A MULTIPLE TRELLIS CODED Q^2PSK SYSTEM FOR PERSONAL COMMUNICATION

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***Abstract*— In this paper the performance of Multiple Trellis-Coded Modulation (MTCM) codes for application to a Q^2PSK personal communication system, are investigated. This paper considers, based on the optimum design criteria for the design of MTCM codes over fading channels, the design of new 4-state codes for Q^2PSK. The benefits in using channel state information based on fading channel estimates in conjunction with the MTCM decoder through a suitable metric weighting function are also considered. The channel estimates are obtained from sequences of known header and pilot symbols embedded in the information stream. This technique is shown to provide remarkably robust performance in the presence of fading. The bit error rate performance of the MTCM code designs (including channel side information), assuming coherent detection and perfect carrier and time synchronization, are then studied by means of simulation for cellular radio channels. It is shown that coding gains in excess of 14.0 *dB* at a bit error probability of 10^{-4} can be achieved with the MTCM coded system, relative to the uncoded system on Rayleigh fading channels.**

I. INTRODUCTION

During the last years Personal Communication Systems (PCS) have been experiencing a rapidly growing market, and this trend will increase even more in the near future. The ultimate goal of today's communication engineers are to provide communication services by any person to any person at any place at any time without any delay in any form through any medium. This increasing demand for personal communication services will render the capacity of existing systems inadequate in future. Therefore, modulation techniques utilizing the spectrum more efficiently will have to be adopted for future mobile communications systems.

For the purpose of realizing high spectral and power efficiencies, this paper is primary concerned with the performance of a four-dimensional Quadrature-Quadrature Phase-Shift Keying (Q^2PSK) [1, 2] system in the land and satellite mobile communications environment. Q^2PSK, offering a theoretical bandwidth efficiency of 4.0 *bits/s/Hz*, is a very effective technique for achieving high system capacity. However, precise compensation techniques are also required to offset the severe degradation caused in Q^2PSK systems due to nonlinearities, fast fading and time delay spread present in the mobile communications environment [3]. Fading compensation techniques applicable for traditional two-dimensional systems based on the application of high performance Trellis Coded Modulation (TCM) schemes have been studied in [4, 5]. These trellis codes, together with the transmission of a known pilot sequences embedded in the coded data stream, have shown to effectively compensate for the distortion caused by fast Rayleigh fading [6, 7].

As far as coding is concerned, it is well-known that the appropriate criterion for designing good TCM schemes for the AWGN channel is to maximize the minimum Euclidean Distance (ED) between any two distinct information sequences of the coded sequences. Several studies [4, 5, 8] have shown that the error rate performance of TCM schemes over fading channels can be strongly influenced by the effective or shortest error event path, and the minimum product distance along that error event path. On fading channels, these parameters play a more important role than the minimum ED [8]. In other words, when choosing trellis codes for fading channels, time diversity is of greater importance than asymptotic coding gain.

In previous studies of coding strategies for Q^2PSK, mostly Rician channels have been considered [9, 10]. In this paper the results of [10] will be extended to include the performance on Rayleigh fading channels. Furthermore, the use of Channel State Information (CSI) obtained from a channel estimator is extended by appropriate metric weighting functions to improve Viterbi decoder performance. This improvement depends strongly on the reliability of the CSI.

The rest of the paper is organized as follows: Section 2 describes the multiple trellis coded Q^2PSK transceiver discussed in this paper. In Section 3, based on the optimum design criteria for the design of MTCM codes for fading channels, the design of a specific 4-state rate-6/8 (multiplicity 2) and rate-6/12 (multiplicity 3) MTCM codes are carried out. Following this, the use and benefits of the CSI provided by the inserted pilot symbols are investigated. Section 4 contains computer simulations of the Bit Error Rate (BER) performance results for Q^2PSK PCS under typical radio channel conditions. The paper is concluded in Section 5.

II. TRANSCEIVER DESCRIPTION

The block diagram of the Q^2PSK PCS under investigation is shown in Fig. 1(a). The input binary stream is first passed through the MTCM encoder and signal-point mapper. After mapping successive blocks of 4 bits into the 16-point Q^2PSK

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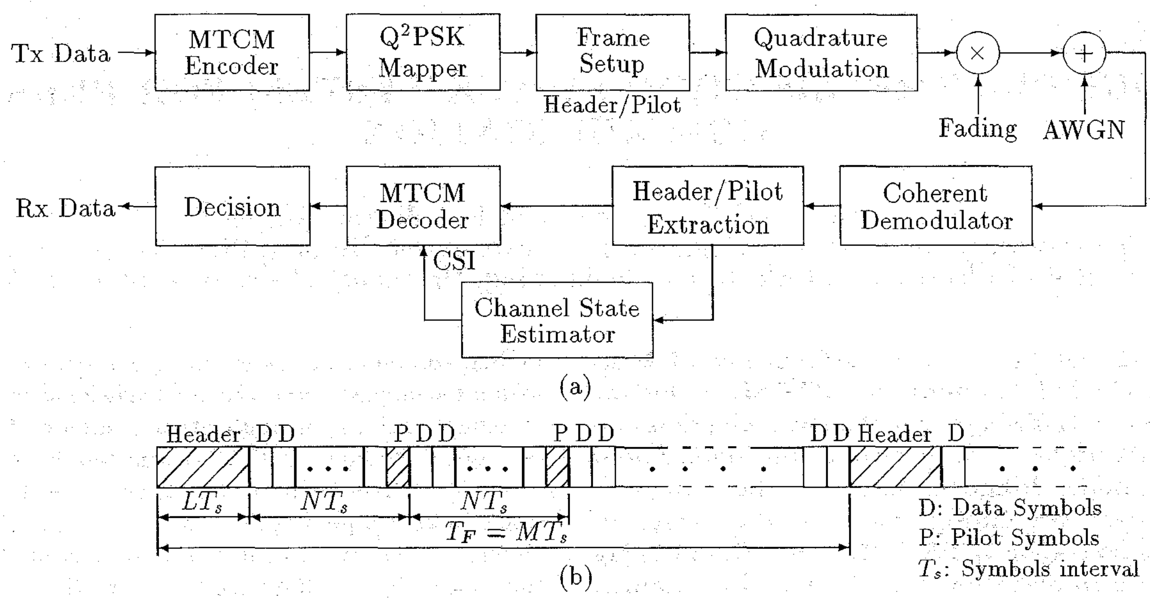

Fig. 1. *Q^2PSK PCS (a) System block diagram (b) Frame transmission strategy, with header/pilot symbol insertions.*

constellation, the known reference headers and pilot symbols are inserted. The position of the pilot symbols within the frame has an insignificant effect on bit error performance [6]. The reference header of length L symbols (known to the receiver) is utilized to establish synchronization (time, clock, and symbol), and also to indicate the state of the fading during the header transmission. In addition to this a known pilot symbol is inserted in every frame of length N symbols. This frame transmission strategy is illustrated in Fig. 1(b), with MT_s the total frame duration.

After header/pilot symbol insertion, the I (in-phase) and Q (quadrature) baseband signals are quadrature modulated onto a carrier frequency of f_c. The modulated signal is given by

$$\begin{aligned} s_T(t) &= \sqrt{2}\{s_I(t)\cos(2\pi f_c t) - s_Q(t)\sin(2\pi f_c t)\} \\ &= \Re\left[z_T(t)\, e^{j2\pi f_c t}\right] \end{aligned} \tag{1}$$

where

$$z_T(t) = \sqrt{2}\,[s_I(t) \;+\; js_Q(t)] \tag{2}$$

is commonly referred to as the complex envelope of the transmitted signal, and the additional $\sqrt{2}$ is a normalizing factor.

In a mobile radio channel, the received signal is a linear combination of a large number of carrier signals spread in time and frequency, each corrupted by AWGN. In relatively low symbol rate systems (e.g., $R_s < 50$ *kSymbols/s*), the time delay spread among these signal paths is frequently a negligible fraction of the symbol interval T_s. For PCS in a micro-cellular radio system, the fading effects can be modeled as being non-frequency selective (or flat fading), with a Rayleigh amplitude and uniform phase distribution [11]. These channels are characterized by the Rician parameter, K, and the product of the maximum Doppler frequency, f_D, and the symbol duration, T_s.

The complex envelope of the faded carrier $u(t)$ may be represented as

$$u(t) = a(t)\; z_T(t) \tag{3}$$

where the quantity $\alpha(t) = A(t)\; e^{j\,\theta(t)}$ represents the fading. $A(t)$ and $\theta(t)$ are, respectively, the fading amplitude and phase processes.

The received signal is coherently demodulated with a locally generated carrier reference, assuming perfect carrier and time synchronization. The demodulated and low-pass filtered complex signal is given by

$$s_R(t) = \alpha(t)\; z_R(t) + n(t) \tag{4}$$

Here, $z_R(t)$ is the signal component of the received complex baseband signal, and $n(t)$ is a independent, zero mean white Gaussian noise process. The distortion caused by fading is represented by the complex function $a(t)$. The inserted header symbols are utilized in the channel estimator to derive an estimate of the fading amplitude and phase over the header interval. This estimate is used as CSI in the MTCM decoder, and is constantly updated by the extraction the fade estimate dividing $s_R(t)$ by the corresponding transmitted pilot symbol.

During the decoding process, the MTCM decoder takes into account the state of the fading channel by means of CSI based metric weighting functions. This scheme assumes the channel to be unreliable when deeply faded and gives lower weights to branch metrics computed during such intervals. After decoding and decision making the decoded bit stream is recovered and delivered to the output.

III. MULTIPLE TRELLIS CODE DESIGN

In the design of the MTCM codes the *Ungerboeck: From Root-to-Leaf* procedure presented by Biglieri et al. [8], has been followed. This procedure makes use of k-fold Cartesian products of the sets found in Ungerboeck's original set-partitioning method for conventional trellis codes [12]. The parameter k is referred to as the *multiplicity* of the code, and it represents the number of Q^2-ary symbols allocated to each branch in the trellis diagram ($k = 1$ corresponds to conventional TCM). Attention is focused on the design of a code with multiplicity, $k = 2$ and 3. Burst error analysis have revealed the chosen multiplicity factors to be sufficient in breaking up on average any correlation between successive symbols, assuming a fast fading channel, characterised by low Rician parameters ($K < 5$ dB), with Doppler frequencies up to 5% of the symbol rate, i.e., $f_D T_s \leq 5 \times 10^{-2}$.

A. Ungerboeck set-partitioning: From Root-to-Leaf

Let A_0 denote the complete Q^2PSK signal set (i.e., signal points $0, 1, \ldots, M-1 = 15$) and $A_0 \otimes A_0$ denote for $k = 2$, the two-fold ordered Cartesian product of A_0 with itself. For a code multiplicity of 3, the process is initiated with the three-fold Cartesian product set, $A_0 \otimes A_0 \otimes A_0$).

Considering the code design for multiplicity, $k = 2$, the first step is to partition $A_0 \otimes A_0$ into M signal sets defined by the ordered Cartesian product $\{A_0 \otimes B_i\}$, $i = 0, 1, \ldots, M-1$. The second element $\{j_2\}$ of B_i is defined by $nj + i \ mod \ M$. Since the squared ED between any pair of 2-tuples is the sum of the distances between corresponding symbols in the 2-tuples, the set partitioning guarantees that the *intradistance* (i.e., distance between pairs within a specific set or partition) of all of the partitions $A_0 \otimes B_i$ is identical. For this set, the minimum product of squared distances over all pairs of 2-tuples in $A_0 \otimes B_0$, $\prod d_{ij}^2$, must be maximized. This is done by choosing the odd integer multiplier, n such that it produces a *maximin* solution. A computer search for possible values of n, revealed the solution to be $n = 11$. The generating sets, $A_0 \otimes B_0$ for $k = 2$, and $A_0 \otimes B_0 \otimes A_0$ for $k = 3$, are illustrated below for Q^2PSK with $M = 16$.

$$A_0 \otimes B_0 = \begin{bmatrix} 0 & 0 & 8 & 8 \\ 1 & 11 & 9 & 3 \\ 2 & 6 & 10 & 14 \\ 3 & 1 & 11 & 9 \\ 4 & 12 & 12 & 4 \\ 5 & 7 & 13 & 15 \\ 6 & 2 & 14 & 10 \\ 7 & 13 & 15 & 5 \end{bmatrix} \quad (5)$$

$$A_0 \otimes B_0 \otimes A_0 = \begin{bmatrix} 0 & 0 & 0 & 8 & 8 & 8 \\ 1 & 11 & 1 & 9 & 3 & 9 \\ 2 & 6 & 2 & 10 & 14 & 10 \\ 3 & 1 & 3 & 11 & 9 & 11 \\ 4 & 12 & 4 & 12 & 4 & 12 \\ 5 & 7 & 5 & 13 & 15 & 13 \\ 6 & 2 & 6 & 14 & 10 & 14 \\ 7 & 13 & 7 & 15 & 5 & 15 \end{bmatrix} \quad (6)$$

The remaining sets obtained by this first partition can easily be derived, each with having a minimum intradistance of $8.0E_b$.

B. Code design

In order to realize a rate-6/8 trellis code of multiplicity $k = 2$, two Q^2PSK symbols are transmitted over the channel for each 6 bits accepted by the encoder. Consequently, the number of branches associated with each state (i.e., emanating from or terminating in a node) equals $2^6 = 64$. Dividing this by the number of states, i.e., 4 in this case, a cardinality of not fewer than 16 symbols is required. Following a similar approach, a rate-6/12 trellis code, with code multiplicity $k = 3$ was implemented. The trellis diagram of these fully-connected trellis codes with cardinality of 16 is shown in Fig. 2, where $A, \ldots, H$ are chosen as those sets that have the largest *interdistance*.

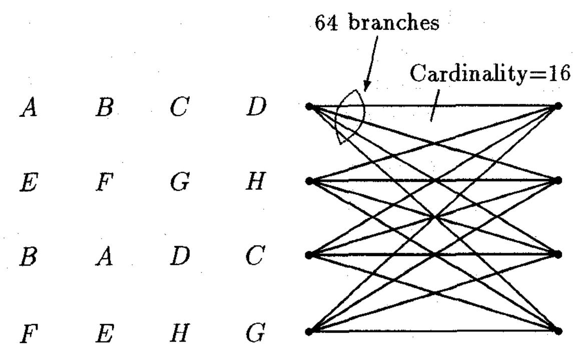

Fig. 2. *Trellis diagram for fully-connected 4-state rate-6/8 ($k = 2$) and rate-6/12 ($k = 3$) multiple trellis codes.*

Thus, for $k = 2$ the sets are given by:

$$\begin{aligned} A &= A_0 \otimes B_0, & B &= A_0 \otimes B_3 \\ C &= A_0 \otimes B_5, & D &= A_0 \otimes B_6 \\ E &= A_0 \otimes B_7, & F &= A_0 \otimes B_9 \\ G &= A_0 \otimes B_{10}, & H &= A_0 \otimes B_{11} \end{aligned} \quad (7)$$

and for $k = 3$, the sets are given by:

$$\begin{aligned} A &= A_0 \otimes B_0 \otimes A_0, & B &= A_0 \otimes B_3 \otimes A_0 \\ C &= A_0 \otimes B_5 \otimes A_0, & D &= A_0 \otimes B_6 \otimes A_0 \\ E &= A_0 \otimes B_7 \otimes A_0, & F &= A_0 \otimes B_9 \otimes A_0 \\ G &= A_0 \otimes B_{10} \otimes A_0, & H &= A_0 \otimes B_{11} \otimes A_0 \end{aligned} \quad (8)$$

all with minimum interdistance between any set equal to $8.0E_b$.

C. Decoder weighting functions

Assume that symbol- and frame synchronization have been achieved. Since the position and times of occurrence of the sequences of pilot symbols are known to the receiver, $\alpha(t)$ in (4) can be estimated from these symbols and is subsequently used as CSI in the modified Viterbi decoder. If the pilot symbol spacing is small enough, a linear extrapolation technique (described in [7]) can be used to estimate the fading channel state of the data symbols succeeding the pilot symbol. Due to the noise effects, perfect estimation is not possible.

In the implementation of the Viterbi algorithm in the MTCM decoder a modified branch metric is defined which uses a weighting function. This modified Viterbi algorithm includes the knowledge of the channel state obtained from the header and pilot sequences. Mathematically, the decoding algorithm is required to determine the sequence of code symbols $\{c_1, c_2, \ldots, c_m\}$ which minimizes

$$\sum_{i=1}^{m} | \hat{\alpha}_i |^2 [r_i - c_i]^2 \tag{9}$$

over the set of allowable code sequences. r_i is the ith received code sample, and $\hat{\alpha}_i$ is the ith channel estimate. The impact of the CSI as well as that of channel estimation errors passed to the decoder by this weighting function will be demonstrated through simulation results. Note that, for the case of no decoder weighting, the fading estimate α_i is unity.

IV. PERFORMANCE EVALUATION

This section presents computer simulation results of the uncoded and coded Q^{2}PSK PCS. For all the simulations, a frame length M of 80 symbols, header length L of 8 symbols, and sub-frame length N of 8 symbols were assumed. M was also taken as the decoding depth of the Viterbi decoder. The effect of varying M and N is left for further study.

Figures 3 and 4 show the BER results of the MTCM Q^{2}PSK system, compared to the uncoded system performance under fast Rayleigh fading conditions. The performance of the uncoded Q^{2}PSK system under ideal conditions in AWGN is also presented. The BER performance curves are shown for values of normalized Doppler frequency $f_D T_s$ equal to 0.004, corresponding to a symbol rate of 5 *kSymbols/s*, a RF carrier frequency of 900 *MHz*, and a vehicle speed of 120 *km/h*.

Fig. 3 shows the BER performance of the coded system in Rician ($K = 5$ *dB*) fading, compared to that of the uncoded system. The BER curves with and without channel state estimation and weighting in the decoder are provided. Comparing the performance of the diversity $k = 2$ (rate 6/8) and 3 (rate 6/12) MTCM systems an improvement in excess of 10 *dB* is obtained, compared to the uncoded (rate 1/1) system under identical conditions. The performance of the $k = 2$ and 3 decoding systems with and without channel estimation are comparable. These results demonstrate that under these channel situations, the multiplicity of the code does not constitute the primary objective for optimum performance. This is true since the free ED of the code dominates the performance. At low values of E_b/N_o the presence of CSI provides little improvement. This is due to the inability of the estimation/compensation scheme to track the rapid fading accurately as a result of the noise. However, the utilization of CSI leads to an improved BER performance as the E_b/N_o ratio increases.

The BER performance of the Q^{2}PSK PCS on the Rayleigh fading channel are shown in Fig. 4. The BER performance

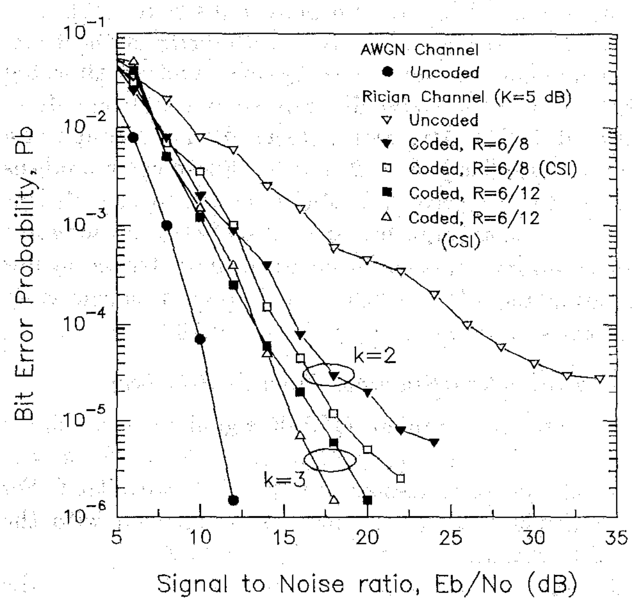

Fig. 3. *BER results for 4-state rate-6/8 and rate-6/12 multiple trellis coded Q^2PSK PCS operating on Rician ($f_D T_s = 0.004, K = 5$ dB) fading channel.*

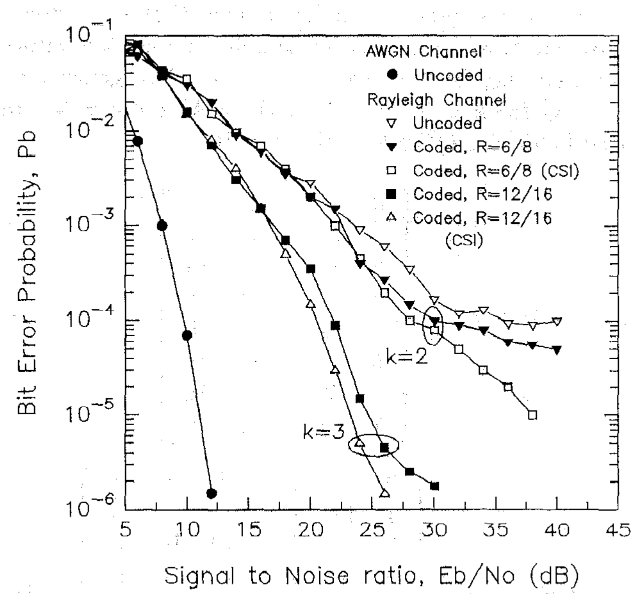

Fig. 4. *BER results for 4-state rate-6/8 and rate-6/12 multiple trellis coded Q^2PSK PCS operating on Rayleigh ($f_D T_s = 0.004, K = -\infty$ dB) fading channel.*

Table 1. *Performance comparison of the uncoded and multiple trellis coded Q^2PSK system, operating on the Rician ($K = 5$ dB) and Rayleigh ($K = -\infty$ dB) fading channels at a BER of $P_b = 10^{-4}$ dB.*

Code rate	Code multiplicity	E_b/N_o for $P_b \leq 10^{-4}$	Fading channel	Gain over uncoded Q^2PSK
6/8	$k = 2$	16.1 dB	Rician	10.4 dB
6/8	$k = 2$ (CSI)	14.5 dB	Rician	11.9 dB
6/12	$k = 3$	13.2 dB	Rician	13.3 dB
6/12	$k = 3$ (CSI)	13.2 dB	Rician	13.3 dB
6/8	$k = 2$	30.0 dB	Rayleigh	6.2 dB
6/8	$k = 2$ (CSI)	27.8 dB	Rayleigh	8.4 dB
6/12	$k = 3$	22.1 dB	Rayleigh	14.1 dB
6/12	$k = 3$ (CSI)	20.5 dB	Rayleigh	15.7 dB

of the uncoded and coded ($k = 2$) (with and without channel weighting) systems is severely affected, and irreducible error floors appear. This can be attributed to the combined effects of noise, channel fading and estimation errors relayed to the channel decoder. Furthermore, the choice of the multiplicity factor of 2 is insufficient, and the decoder is therefore unable to cope with the severity of the fading channels considered.

On the other hand, relatively high coding gains are noticed for the third order diversity coded system, both with and without channel weighting. This implies that the choice on the multiplicity ($k = 3$) of the code was sufficient. Also, the coded system employing the CSI provides the best performance in terms of BER improvement. The performance results are summarized in Table 1.

V. CONCLUSIONS

A multiple trellis coding/decoding scheme for Q^2PSK in a fast fading channel environment has been studied. The design criteria presented in the open literature were utilized in the designs of 4-state MTCM coders of rate-6/8 and 6/12, and multiplicity factors $k = 2$ and 3. Extensive performance evaluation based on computer simulation of the Q^2PSK (uncoded and coded) PCS has been carried out under AWGN and mobile fading channel conditions.

Results on the Rayleigh fading channel have shown the importance of the multiplicity factor of the trellis coded system under these channel conditions. Furthermore, the use of CSI in the decoding process with modified metric has shown to improve Viterbi decoder performance in the case of a coding scheme with third order multiplicity. This improvement depends strongly on the reliability of the CSI. The selection of optimal pilot symbol spacings to maintain system performance over a wide range of fast fading channel conditions, warrants further investigation.

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