On the performance of Super-Orthogonal Turbo-Transmit Diversity for CDMA cellular comunication

Danie van Wyk[†] and Pieter van Rooyen[‡]
[†]CSIR Defencetek, Pretoria, South Africa
e-mail: dvwyk@csir.co.za
[‡]Sony CSL, Tokyo, Japan
e-mail: pieter@csl.sony.co.jp

Abstract — **To achieve maximum theoretical system capacity in a Rayleigh fading environment, it has been shown that a combination of multiple transmit-receive antennas (given ideal channel state information and independent fading between antenna pairs) transmit and receive diversity will lead to the maximum achievable system capacity. In a CDMA environment, where the advantages of low rate coding for spreading purposes can be exploited, a coding-diversity gain, also known as space-time gain, can be achieved. This paper presents the new concept of layered super-orthogonal turbo transmit diversity (SOTTD) for code division multiple access (CDMA) communications. The techniques of spreading and coding at low-rate are married with code-division transmit diversity (CDTD) and iterative "turbo" processing. In layered space-time TC CDTD, a turbo encoder (and its associated iterative decoder) is required for every transmit diversity branch available. In SOTTD a single turbo encoder-decoder pair is required, making it a practically feasible solution. The only requirement being that the number of constituent encoders Z, should be greater or equal to the transmit diversity order M_T. The principle of operation is to transmit the coded bits, stemming from the constituent encoders, via the spatial domain rather than via the time, code or frequency domain. The received data stream is then iteratively decoded using turbo decoding principles. SOTTD performance is compared with TC CDTD using both simulations and theoretical bounds.**

I. Introduction

Receive antenna diversity is a widely applied technique to reduce the detrimental effects of multipath fading in wireless communications. However, it is hard to efficiently use receive antenna diversity at the mobile units since they should remain relatively simple, inexpensive and small. Therefore, receive diversity has been used nearly exclusively at the base station. In order to enable high data rate transmission over multi-access wireless fading channels, recently different transmit diversity techniques have been introduced to benefit from antenna diversity also in the forward link while putting the diversity burden on the base station. In [1, 2, 3] Tarokh *et al.* introduced space-time codes proposing a joint design of coding, modulation and transmit diversity for flat Rayleigh fading channels. By avoiding destructive superposition after combination of the signals transmitted simultaneously from different antennas space-time codes achieves the same, theoretically optimal, diversity advantage as receive diversity. In [4, 5] the techniques of convolutional (CC) and TC code-division transmit diversity (CDTD) were presented for cellular CDMA. It was shown that these space-time codes achieve the maximum possible combined diversity/coding gain when used with orthogonal spreading sequences.

In [6, 7, 8], the superior performance achieved with space-time TC transmit diversity (TTD) systems have been presented. It has been shown that TDD offers a sub-optimal, but practical implementation of the TC CDTD system. In layered space-time TC CDTD, a turbo encoder (and its associated iterative decoder) is required for every transmit diversity branch available. This paper extends the TTD signaling scenario by the introduction of the super-orthogonal turbo-coded transmit diversity (SOTTD) [9]. In SOTTD the techniques of spreading and coding at low-rate are married with the CDTD and iterative "turbo" processing. In layered space-time TC CDTD, a turbo encoder (and its associated iterative decoder) is required for every transmit diversity branch available. In SOTTD only a single turbo encoder-decoder pair is required, making it a practical feasible solution. The only requirement being that the number of constituent encoders Z, should be greater or equal to the transmit diversity order M_T. In very general terms SOTTD can be considered as a special case of orthogonal turbo-coded CDTD, employing codes of very low rate. The principle of operation is to transmit the coded bits, stemming from the constituent encoders, via the spatial domain rather than via the time, code or frequency domain. The received data stream is then iteratively decoded using turbo decoding principles. The way the turbo coder is configured fits naturally into the transmit diversity schemes described above.

II. System Description

A synchronous CDMA mobile communications system is considered where the transmitter is equipped with M_T antennas at the base station and a single receiving antenna at the mobile. The signals on the matrix channel, i.e. the $M_T \times 1$ transmission paths between transmitter and receiver, are supposed to undergo independent frequency selective Rayleigh fading. It is assumed that the path gains are constant during one frame and change independently from one frame to another (quasi-static fading).

1. SOTTD Transmitter

The general structure of the proposed encoding and diversity transmission scheme is illustrated in Fig. 1. Owing to this encoding structure, the encoding procedure is frame oriented. A binary data sequence $\mathbf{b}$ of length N is fed into the

0-7803-6560-7/00/$10.00 (C) 2000 IEEE

encoder. The hart of the encoding scheme is formed by the Z rate-1/16 constituent encoders, consisting of the combination of a rate-1/4 recursive systematic convolutional (RSC) encoder and a rate-4/16 WH orthogonal modulator, denoted by (RSC&WH). These encoders are concatenated in parallel applying an interleaver. The first encoder processes the original data sequence, whereas before passing through the Zth encoder, the data sequence is permuted by a pseudo random interleaver of N. The outputs of the Z constituent encoders are punctured in order to provide a wide range of code rates.

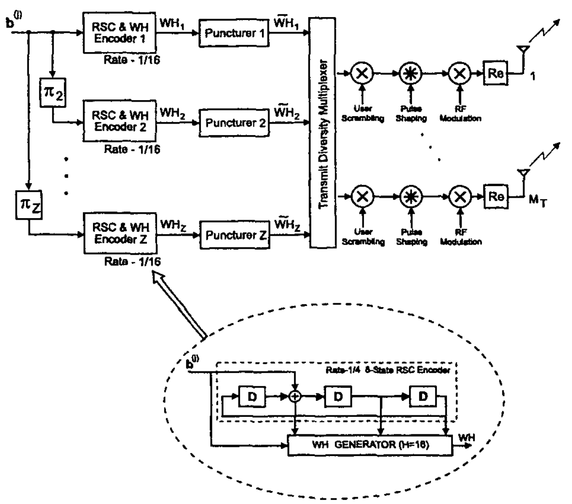

Fig. 1: Transmitter block diagram.

The detailed structure of the combined RSC turbo and WH encoder is also shown in Fig. 1. The state outputs of the rate-1/4 RSC encoder is fed to the rate-4/16 WH, producing one sequence of length ($L_{WH} = 16$) from a set of $H = 2^4 = 16$ sequences. The generator matrix for the $L_{WH} = 16$ WH in systematic form is given as

$$G_{WH} = \begin{bmatrix} 1000\ 011011010101 \\ 0100\ 010110110011 \\ 0010\ 001110001111 \\ 0001\ 000001111111 \end{bmatrix}$$

Recall from coding theory, that the most important characteristic of a codeword is its minimum free distance. Owing to the orthogonality characteristics of the WH codewords, the minimum distance of the encoded sequences for both constituent encoders is equal to $d^{WH} = L_{WH}/2 = 8$. In addition, full-rank transmit diversity may be achieved provided that the transmit antennas are sufficiently spaced.

After encoding, the output sequences are obtained by appropriate puncturing according to puncturing patterns $P^{(i)} = \{p_1^i, p_2^i, \ldots, p_N^i\}$, where $i = 1$ and 2, for the first and second puncturer, respectively. With $W_{P^{(1)}}$ and $W_{P^{(2)}}$, the weights of the first and second puncturers, respectively, the resulting overall encoder rate (R_c) is given by:

$$R_c = \frac{1}{W_{P^{(1)}} + W_{P^{(2)}}} \tag{1}$$

Therefore, for the case when none of the output sequences' bits are punctured the overall code rate of the combined turbo and WH encoding strategy is given as $R_c = 1/(16+16) = 1/32$.

After encoding, the Z encoded streams are multiplexed to the M_T available transmit antenna section, encapsulating the user specific scrambling, spreading and chip shaping.

2. SOTTD Receiver and Iterative Decoder

For description a dual transmit, $M_T = 2$ and single receive antenna, $M_R = 1$ system is assumed. Without loss of generality, the number of constituent encoders Z is taken as 2, i.e. $M_T = Z = 2$. Fig. 2 shows general receiver for the SOTTD system, as well as the iterative turbo decoding strategy.

Before the actual decoding takes place, for those bits that were punctured, zero values are inserted. Therefore the decoder regards the punctured bits as erasures. The iterative decoding of the turbo coding scheme requires two component decoders using soft inputs and providing soft outputs. The soft output Viterbi algorithm (SOVA) or maximum *a posteriori* (MAP) algorithm may be employed.

Detail concerning the actual decoding process will now be given, with reference to Fig. 2.

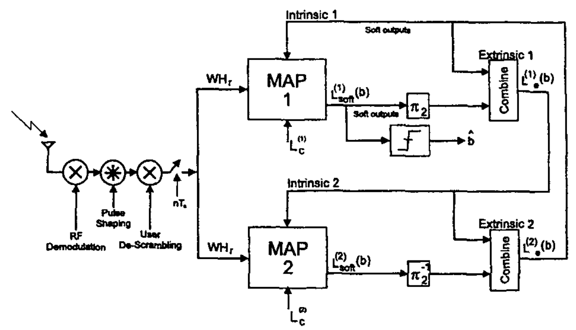

Fig. 2: Receiver block diagram.

It is assumed for the analysis, that none of the encoded sequences' bits are punctured (i.e., $WH_z = \tilde{WH}_z$, where $i = 1, \ldots, Z$).

Let WH_r be the associated received and demodulated branch with the corresponding reliability values of the channel, $L_c^{(z)}$, $z = 1, 2$, depending on whether decoder 1 or 2 is being used. The decoder accepts a priori values $L_i(b)$ for all the information bit sequences and soft-channel outputs $L_c^{(z)} \cdot WH_r$

The branch metric calculation is performed very efficiently by using soft-output inverse WH (SO-IWH) transformation, which basically correlates the received WH sequence with the branch WH sequences terminating in a specific node. Then, by discarding the branch with the lowest accumulated path metric, the maximum likelihood branch is retained.

The soft-input soft-output delivers *a posteriori* soft outputs $L(\hat{b})$ for all the information bits and extrinsic information $L_e(b)$, which for the current bit is only determined by its surrounding bits and the code constraints. It is therefore independent of the intrinsic information and the soft output values of the current bit. It is important to note that all the above mentioned sequences are vectors of length $L_{WH} = 16$.

Ideally, the log-likelihood ratio (LLR) soft-output of the decoder for the information bit b is written as

$$L(\hat{b}) = (L_c \cdot WH_r + L_i(b)) + L_e(\hat{b}) \qquad (2)$$

implying that there are three independent estimates, which determine the LLR of the information bits: the a priori values, $L_i(b)$, the soft-channel outputs of the received sequences $L_c \cdot WH_r$, and the extrinsic LLR's $L_e(\hat{b})$.

At the commencement of the iterative decoding process there usually are no a priori values $L_i(b)$, hence the only available inputs to the first decoder are the soft-channel outputs obtained during the actual decoding process.

After the first decoding process the intrinsic information on b is used as independent[1] *a priori* information at the second decoder. The second decoder also delivers *a posteriori* information, which is used to derive the extrinsic information, which is - for its part - used in the subsequent decoding process at the first decoder in the next iteration step. The final decision is, of course, based on the *a posteriori* information, output from the second decoder. Note, that initially the LLRs are statistically independent, however, since the decoders use indirectly the same information, the improvement through the iterative process becomes marginal, as the LLR's become more and more correlated.

3. Points to Ponder

Various authors have stated that the design of a optimal interleaver helps to avoid low weights of encoded sequences in many cases, which leads to improved bit error rate performance. In our context, the weight of the encoded sequences (excluding the all-zeros codeword) also equals $d^{WH} = 8$. This is an important observation since it removes the requirement to design an optimal interleaver. For this reason a simple pseudo random interleaver is utilized.

The size of the interleaver, however, and not its design (for our application), determines the performance of the coded system. The larger the interleaver size (N), the larger the "interleaver gain" and greater the potential to increase the temporal spread of successive bits of the original data sequence.

It is important to note that the constituent RSC&WH encoders may produce similar WH codewords. Since these codewords are transmitter over different antennas the full-rank characteristic of the system is still guaranteed. Under multipath fading scenarios, some of the orthogonality will be destroyed. The latter is not a function of the specific WH codeword transmitted at the different antennas, but rather dependent on the delay spread of the channel. Transmitting the same WH codewords over the different antennas will have an effect on the channel estimation and initial system synchronization procedures.

The performance of the SOTTD system depends not on the distance properties of the WH code, but actually on the distance properties of the combined RSC&WH code. In this context, the most important single measure of the code's ability to combat interference is d_{min}.

Fig. 3 depicts the modified state diagram of the RSC&WH constituent code under consideration. The state diagram provides an effective tool for determining the transfer function, $T(L, I, D)$, and consequently d_{min} of the code. The exponent of D on a branch describes the Hamming weight of the encoder

[1]Interleaving between the two decoders reduces the statistical dependencies effectively.

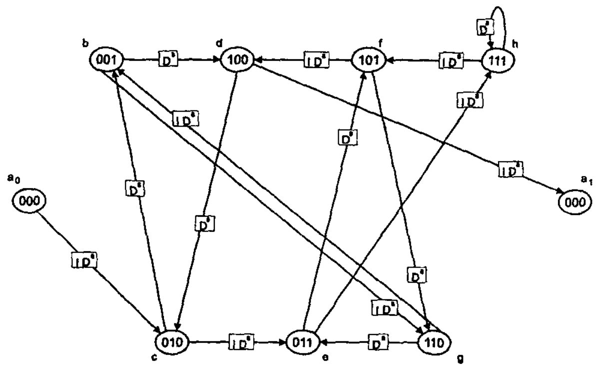

Fig. 3: Trellis diagram of RSC&WH constituent encoder.

corresponding to that branch. The exponent of I describes the Hamming weight of the corresponding input word. L denotes the length of the specific path.

Through visual inspection the minimum distance path, of length $L = 4$ can be identified as: $a_0 \to c \to b \to d \to a_1$. This path has a minimum distance of $d_{min} = 4 \times d^{WH} = 32$ from the all-zero path, and differs from the all-zero path in 2 bit inputs.

III. Performance Evaluation

In this section it is attempted to shed some light on the theoretical comprehension of parallel concatenated WH codes. In particular, an upper bound to the average performance of the parallel concatenated codes, stemming from characteristics of the combined RSC&WH (where $L_{WH} = 32$) constituent code, will be defined and evaluated.

Given an (n, k) RSC&WH constituent code, C_z, its input redundancy weight enumerating function (IRWEF) is given by [10]:

$$A^{C_z}(I, D) \doteq \sum_{i,d_p} A_{i,d_p} I^i D^{d_p}, \qquad (3)$$

where A_{i,d_p} is the integer number of codewords generated by an input word with Hamming weight i whose parity check bits have Hamming weight d_p. Therefore, the overall Hamming weight is $d = i + d_p$. The IRWEF characterizes the whole encoder, as its depends on both the input information words and codewords.

The IRWEF makes implicit in each term of the normal weight enumerating function (WEF) the separate contributions of the information and the parity-check bits to the total Hamming weight of the codewords.

When the contributions of the information and redundancy bits to the total codeword weight is split, the IRWEF for the constituent WH code is obtained as

$$A^{C_z}(I, D) = 1 + 4ID^7 + 6I^2D^2 + 4I^3D^5 + I^4D^4 \qquad (4)$$

When employing a turbo interleaver of length kN, the IRWEF of the new constituent (nN, kN) code C_z^N is given by

$$A^{C_z^N}(I, D) = \left[A^{C_z}(I, D)\right]^N, \qquad (5)$$

for all Z the constituent codes.

Using (5) the conditional WEF, $A_i^{C_z^N}(D)$ of the constituent codes can be obtained from the IRWEF as

$$A_i^{C_z^N}(D) = \frac{1}{i!} \cdot \frac{\delta^i A^{C_z^N}(I,D)}{\delta I^i}\bigg|_{I=0}. \quad (6)$$

From the conditional WEF, owing to the property of the uniform interleaver of length kN, the conditional WEF of the two-constituent ($Z = 2$) parallel concatenated code of length $((2n-k)N, kN)$ is obtained as

$$A_i^{C_P^N}(D) = \frac{A_i^{C_z^N} \cdot A_i^{C_z^N}}{\binom{kN}{i}}. \quad (7)$$

The IRWEF of the parallel concatenated code using the the following inverse relationship can be obtained as

$$A^{C_P^N}(I,D) = \sum_i I^i A_i^{C_P^N}(D). \quad (8)$$

To compute an upper bound to the bit error probability (BEP), the IRWEF can be used with the union bound assuming maximum likelihood (ML) soft decoding. The BEP, including the fading statistics (assumed to be slow fading), assumes the form

$$P_{b|S} \leq \frac{I}{k} Q\left(\sqrt{d_{min}\sigma_{0c}\mathrm{s}}\right) \cdot e^{d_{min}\sigma_{0c}\,\mathrm{s}} \cdot \frac{\delta A(I,D)}{\delta I}\bigg|_{I=D=e^{-\sigma_{0c}\mathrm{s}}}, \quad (9)$$

where σ_{0c} denotes the effective signal-to-noise ratio (SNR), and S denotes the power of the received signal.

Assuming that the cellular system is employing omnidirectional antennas, the total output SNR term used in (9) can be determined as

$$\sigma_{0c} = \left(\frac{1}{R_c}\frac{N_o}{2\,E_b} + \frac{(K \cdot M_T - 1)}{3N}\right)^{-1}. \quad (10)$$

Also, if it is assumed that the M_T transmit diversity transmissions are equal powered, with constant correlation between the branches, and transmitted over a Rayleigh fading channel, the components of the received power vector S are identically distributed, with pdf given by

$$p_S(\mathrm{s}) = \frac{1}{\Omega^2\Gamma(M_T \cdot L_R)}\left(\frac{\mathrm{s}}{\Omega^2}\right)^{M_T \cdot L_R - 1} \times \frac{\exp\left(-\frac{\mathrm{s}}{(1-\rho)\Omega^2}\right) \cdot {}_1F_1\left(1, M_T \cdot L_R, \frac{\rho M_T \cdot L_R \mathrm{s}}{\zeta(1-\rho)\Omega^2}\right)}{\zeta\,(1-\rho)^{(M_T \cdot L_R - 1)}}, \quad (11)$$

with $\zeta = 1 - \rho + \rho M_T \cdot L_R$.

From (11), ${}_1F_1(\cdot)$ is the confluent hyper geometric function, Ω^2 is the average received path strength (assumed equal), ρ the correlation between transmit or receive branches, and L_R is the number of RAKE receiver fingers.

Using a finite number of terms in (9) transforms the upper bound into the approximation

$$P_{b|S} \approx \sum_m D_m Q\left(\sqrt{m\sigma_{0c}\,\mathrm{s}}\right), \quad (12)$$

Acronym	Definition	Conditions
Uncoded	Uncoded system	$N_{spread} = N = 32$
CC	Convolutional Coder	S=256, $R_c = 1/2$ $N_{spread} = N/2$
TC	Turbo Coder	S=4, $R_c = 1/2$ $N_{spread} = N/2$
SOTC	Super-Orthogonal Turbo Coder	S=8, $R_c = 1/32$ $N_{spread} = N/32$
CDTD	Code-Division Transmit Diversity	Uncoded, CC/TC $M_T = 2, 3$
SOTTD	Super-Orthogonal TTD	SOTC $M_T = 2$

Table 1: Summary of Techniques.

where

$$D_m = \sum_{i+d_p=m} \frac{i}{k} A_{i,d_p}, \quad (13)$$

where A_{i,d_p} is obtained from the IRWEF of parallel concatenated code (compare (3)).

Finally, the BEP is computed using (12) and (13), when averaged over the fading statistics.

IV. Analytical and Simulation Results

The performance of the ($M_T = Z = 2$) super-orthogonal transmit diversity (SOTTD) CDMA system is compared to that of an uncoded, and convolutional- and turbo coded code-division transmit diversity (CDTD) CDMA systems. Table 1 presents a summary of the techniques of importance in the performance evaluation. For a description of the CC and TC CDTD techniques the reader is referred to [11].

Using the system parameters outlined in Table 2, the BEP performance of a cellular CDMA system employing the different techniques has been determined numerically. The results are shown in Fig. 4.

Parameter	Simulation value
Spreading sequence length	$N = 32 \times R_c$
Operating environment	2-Path, equal strength
User distribution	uniform
Number of MP signals	$L_p = 2$
Number of users	$K = 1, 2, \ldots, N$
Number of RAKE fingers	$L_R = 2$
FEC code type and rate	CC, TC ($R_c = 1/2$) SOTC ($R_c = 1/32$)
Turbo Interleaver Length	$N = 256$
TD elements	$M_T = 1, 2, 3$
TD technique	CDTD and SOTTD

Table 2: System parameters for numerical evaluation of BEP performance.

Shown on the figure are the performance of single and $M_T = 2, 3$ transmit diversity systems' performance. From the curves it is clear that the superior performance predicted for TC CDTD may achieved with the SOTTD system over

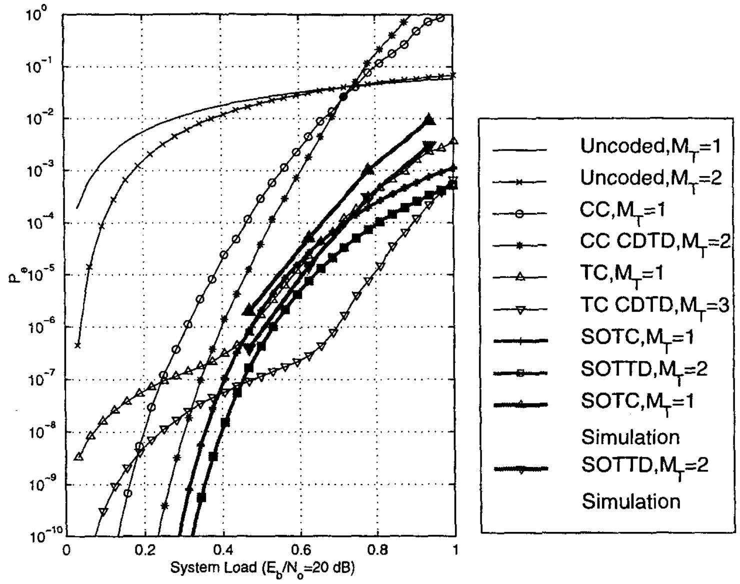

Figure 4: Bit error probability as a function of the load (number of users/total spreading), with operating point of $E_b/N_o = 20$ dB.

the complete capacity range. Also of importance is the fact that the performance degradation of TC CDTD at low system loads (due to inherent TC error floor), is alleviated by the SOTC system, therefore the superior performance of SOTTD. This is explained in terms of the higher minimum free distance on offer by the rate-1/16 constituent encoders, as opposed to the use of rate-1/2 constituent encoders in TC systems.

V. Conclusions

We have presented the a SOTTD signaling scenario for spread-spectrum CDMA communication systems. It provides a very powerful and practical extension to the TC CDTD schemes, providing superior performance over the complete capacity range of CDMA.

In conclusion, there are a number of important notes which must be made about the performance bounds presented in this paper. These bounds are upper limits on the performance of the codes derived from the use of the union bound. As such the bounds are only valid for the case of ML decoding, and they will diverge significantly from the true performance at low values of E_b/N_o or at high system loads. Also, in the simulation a sub-optimal decoding algorithm is used which is not ML.

References

[1] V. Tarokh, A. Naguib, N. Seshadri, and A. R. Calderbank, "Low-rate multi-dimensional space-time codes for both slow and rapid fading channels," in *PIMRC'97: International Symposium on Personal Indoor and Mobile Radio Communications*, (Helsinki, Finland), pp. 1206–1210, September 1997.

[2] N. Seshadri, V. Tarokh, and A. R. Calderbank, "Space-time codes for wireless communications: Code construction," in *VTC'97: Vehicular Technology Conference*, pp. 637–641, 1997.

[3] V. Tarokh, N. Seshadri, and A. R. Calderbank, "Space-time codes for high data rate wireless communication: Performance criterion and code construction," *IEEE Transactions on Information Theory*, vol. 44, pp. 744–765, March 1998.

[4] D. J. van Wyk, I. J. Oppermann, and L. P. Linde, "Low rate coding considerations for space-time coded DS/CDMA," in *IEEE VTC'99: Vehicular Technology Conference*, (Amsterdam, The Netherlands), pp. 2520–2524, September 1999.

[5] D. J. van Wyk, I. J. Oppermann, and L. P. Linde, "Performance tradeoff among spreading, coding and multiple-antenna transmit diversity for capacity space-time coded DS/CDMA," in *Proceedings IEEE MILCOM*, no. 14-1, (Atlantic City, U.S.A), November 1999.

[6] D. J. van Wyk and L. P. Linde, "Turbo-coded/multi-antenna diversity combining scheme for DS/CDMA systems," in *Proceedings of ISSSTA '98*, (Sun City, South Africa), pp. 18–22, September 1998.

[7] D. J. van Wyk, L. P. Linde, and P. G. W. Van Rooyen, "On the performance of a turbo-coded/multi-antenna transmission diversity scheme for DS/CDMA systems with adaptive channel estimation," in *ICT'99: International Conference on Telecommunications*, (Cheju, Korea), pp. 532–537, June 1999.

[8] D. J. van Wyk and L. P. Linde, "Fading correlation and its effect on the capacity of space-time turbo coded DS/CDMA systems," in *Proceedings IEEE MILCOM*, no. 18-4, (Atlantic City, U.S.A), November 1999.

[9] D. J. van Wyk and P. G. W. van Rooyen, "Super-orthognal turbo-coded transmit diversity." SONY Laboratories Preliminary Patent, (01 February 2000), 2000.

[10] S. Benedetto and G. Montorsi, "Unveiling turbo codes: Some results on parallel concatenated codings schemes," *IEEE Transactions on Information Theory*, vol. 42, pp. 409–428, March 1996.

[11] P. van Rooyen and M. Lötter and D. van Wyk, "Space-Time processing for CDMA mobile communications," *Kluwer Academic Publishers*, February 2000.