A Fuzzy Logic Interference Suppression Algorithm for DS-SS Communications

C. Naidoo and P. G. W van Rooyen

Abstract—New techniques to suppress narrow-band and multiple-access interference for Spread-Spectrum (SS) systems are continually being researched for use in the growing cellular CDMA market. This paper investigates the feasibility of using a fuzzy logic technique to adapt the MMSE detector for the suppression of Narrow-band Interference (NBI). The Fuzzy Logic Adaptation (FLA) technique is described and its performance is compared to the Least-Mean-Square (LMS) and the Recursive-Least-Square (RLS) adaptive algorithms. It is found that the fuzzy logic technique considered gives a performance improvement over the conventional and the LMS adaptive algorithm, while it fails to match the performance of the RLS adaptive algorithm.

Index Terms—SS, MMSE, NBI, Fuzzy Logic.

I. INTRODUCTION

As the frequency spectrum is being made available for wide-band SS applications by bandwidth management authorities, industries interest in SS systems is increasing. Industries requirements range from Point-to-Point wireless bandwidth-on-demand systems to third generation cellular systems. The increasing competition for restricted spectral resources will result in the deployed systems experiencing interference from both Narrow-Band and Multiple-Access interferers.

The well documented near-far problem of the conventional match filter detector initiated intense research into near-far resistant receivers. The linear Minimum Mean-Square Error (MMSE) multi-user detector was develop for this purpose. Miller in [2] analysed the training duration requirements of the MMSE multi-user detector and one of the many research proposals presented in his paper was to vary the LMS step-size parameter in order to reduce the convergence time of the MMSE detector. This paper considers the feasibility of varying the step-size parameter by using a fuzzy logic technique and evaluates its Bit Error Rate (BER) performance.

An account of the system used for the simulation is given in section II, which includes a brief introduction into to the fuzzy inference system. Section III presents the results of the simulation and Section IV concludes with a summary of the results.

II. SYSTEM DESCRIPTION

For this study, it is assumed that the receiver is both symbol and chip synchronous to the received signal which is given by,

$$r(t) = S(t) + I(t) + N(t) \tag{1}$$

where $N(t)$ and $I(t)$ represents the AWGN signal and the NBI signal, respectively; $S(t)$ represent the transmitted SS signal over a frame length of $2M+1$ and is given by,

$$S(t) = A\sum_{i=-M}^{M} b(i)s(t-iT) \tag{2}$$

where A, $b(i)$ and $s(t)$ are the receive amplitude, information symbol and spreading sequence, respectively. $s(t)$ is assumed to be supported only on the interval [0,T] and with unit energy.

By passing the received signal (1) through a chip-match filter and sampling the output at the chip-rate, the received signal may be represented as a vector in $\Re^N$ where N is the coding gain. The sample output of the chip-match filter for each symbol interval is given by,

$$\mathbf{x} = b\sqrt{P}\mathbf{s} + \mathbf{i} + \mathbf{n} \tag{3}$$

where $\mathbf{x}$ is the receive signal vector in $\Re^N$; $b = \pm 1$ are the equiprobable information symbols; $\mathbf{s}$ is the normalised spreading sequence, i.e. $\mathbf{s}^T\mathbf{s} = 1$; $\mathbf{i}$ is the wide-sense stationary NBI with zero mean; P is the power of the received signal and $\mathbf{n}$ is the AWGN samples, which has zero mean and a $N \times N$ covariance matrix given by $\sigma_n I$.

The match-filter output is then fed into the tapped delay line filter as shown in fig. 1, which then estimates the received information symbol by evaluating,

$$\hat{b} = \mathrm{sgn}(\mathbf{c}^T\mathbf{x}) \tag{4}$$

where $\mathbf{c}$ is chosen to minimise the mean-square error,

$$MSE \overset{\Delta}{=} E\{(\mathbf{c}^T\mathbf{x} - b\sqrt{P})^2\} \tag{5}$$

It can easily be shown that the optimum solution to (5) is given by,

$$\mathbf{c}_{opt} = \mathbf{R}^{-1}\alpha \tag{6}$$

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where $\mathbf{R} = E\{\mathbf{x}\mathbf{x}^T\}$ which is the Hermitian covariance matrix of the matched-filter output and the cross-correlation vector $\alpha = E\{\mathbf{x}d\}$ with d being the desired information symbol $b\sqrt{P}$.

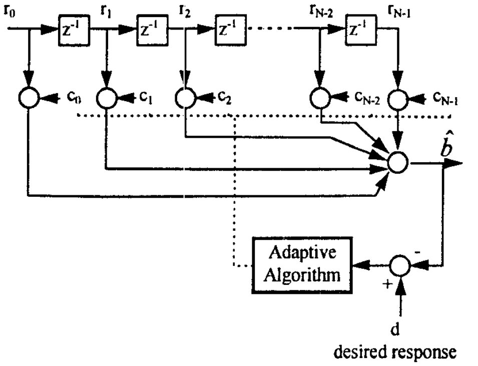

Fig. 1. The N-Tap Adaptive Delay Line Filter

For most applications, the covariance matrix $\mathbf{R}$ is usually unknown. Under these circumstances, the filter coefficients are solve recursively. A brief description of the LMS and RLS algorithms that can recursively solve the filter coefficients is given in the following sections.

A. The LMS Algorithm

The LMS algorithm uses the steepest descent algorithm to iteratively adjust the filter coefficients toward its optimum solution. The coefficient are update at each symbol interval by using the following recursive relation,

$$\mathbf{c}_{k+1} = \mathbf{c}_k - \Delta \mathbf{x}_k e_k \tag{9}$$

where $\mathbf{x}_k e_k$ is the unbiased but noisy estimate of the gradient. To prevent the closed loop from becoming unstable, the step-size is limited to the range [3],

$$0 < \Delta < 2 / N\rho\bar{\lambda} \tag{10}$$

where ρ is the ratio of the maximum eigenvalue to the effective eigenvalue and $\bar{\lambda}$ is the average effective eigenvalue of $\mathbf{R}$. The initial step-size resulting in the fastest convergence of the LMS algorithm is given by[3],

$$\Delta_0 = 1/(N\rho\bar{\lambda}) \tag{12}$$

B. The RLS algorithm

Unlike the MMSE criterion used for the LMS algorithm, the RLS algorithm uses a performance index that is expressed in terms of time averaging instead of statistical or ensemble averaging. For the exponentially windowing approach, the performance index is given by,

$$\frac{\min}{\mathbf{c}_k} \sum_{m=0}^{k} \left\| d(m) - \mathbf{c}_k^T \mathbf{x}_m \right\|^2 \nu^{k-m} \tag{13}$$

where ν is the convergence factor and is in the range $0 < \nu < 1$. The solution to (13) can be computed recursively by the following well known Direct RLS algorithm,

$$e_k = d_k - \mathbf{c}_{k-1}^T \mathbf{x}_k, \tag{14}$$

$$\mathbf{S}_k = \frac{-\mathbf{R}_{k-1}^{-1}\mathbf{x}_k}{\nu + \mathbf{x}_k^T \mathbf{R}_{k-1}^{-1} \mathbf{x}_k}, \tag{15}$$

$$\mathbf{c}_k = \mathbf{c}_{k-1} - e_k \mathbf{S}_k, \tag{16}$$

$$\mathbf{R}_k^{-1} = \frac{1}{\nu}\left[\mathbf{R}_{k-1}^{-1} + \mathbf{S}_k \mathbf{x}_k^T \mathbf{R}_{k-1}^{-1}\right]. \tag{17}$$

C. Description of the FLA Technique

The Fuzzy Inference Systems (FIS) has been widely used in applications were precise knowledge of the inputs to the system is not a prerequisite for a reasonable system decision at the output. The proposed FIS is shown in Fig. 2, where system has a single input, *mmse* and an output, *alph*.

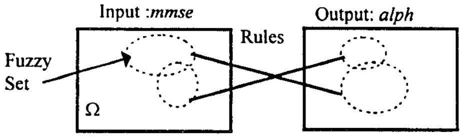

Fig. 2. Fuzzy Set Description of FIS *flmsb*

Every input to the fuzzy system must be a crisp numerical value that is limited to its universe of discourse Ω. The membership functions shown in Figures 3 and 4 defines the fuzzy sets indicated in Fig. 2. If more than one input fuzzy set maps to an output fuzzy set then the AND or OR operations are performed by computing $\min(\mu_A(n), \mu_B(n))$ or $\max(\mu_A(n), \mu_B(n))$, respectively. Where $\mu_X(n)$ is the membership function describing set X. The *mmse low* membership function shown in Fig. 3 was designed to force the MSE to converge quickly by choosing a large step-size for a low MSE input level. While, the *mmse high* membership function attempts to introduce stability by choosing a small step-size for a high MSE level. One would expect the MSE to have a Chi-Square distribution for a interference free SS signal received in AWGN. However, this distribution is difficult to determining due to the continuous adaptation of the filter coefficients. For simplicity, the *mmse* membership functions were chosen to be truncated Gaussian curves as shown in Fig.3

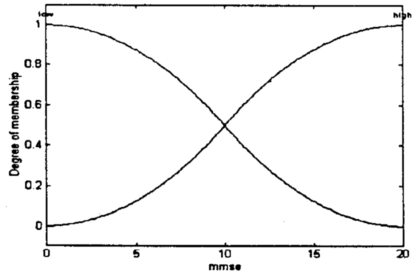

Fig. 3. Input Membership function of FIS *flmsb*

The output membership functions, *alph low* and *high*, were chosen to be symmetrical and triangular. The triangular membership function allows *alph* to be uniformly supported over Ω.

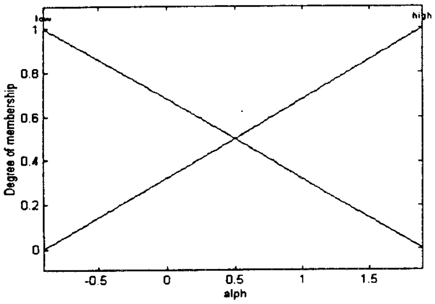

Fig. 4. Output membership function of FIS *flmsb*

The input and output fuzzy sets are link by IF THEN rules which are evaluated using fuzzy reasoning and is given by,

1) *if mmse is low then alph is high,*
2) *if mmse is high then alph is low.*

The corresponding fuzzy *output surface* is shown in fig. 5.

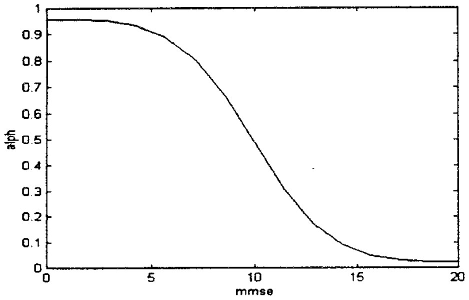

Fig. 5. *Output surface* for FIS *flmsb*

Using the recommendation given in [2], the following relation can be used to recursively computed the step-size using the output of the FIS,

$$\Delta_{k+1} = {alph_k}/{\rho N Pa_k} \tag{18}$$

where $Pa_k = \frac{1}{N}\sum_{i=k}^{k-N} \mathbf{x}_i \mathbf{x}_i^T \approx \bar{\lambda}$ which is an approximation of the average effective eigenvalue due to $\mathbf{R}$ consisting of only $1/N$ significant eigenvalues. By the same token, $\rho\bar{\lambda} \approx \lambda_{\max}$ so that (18) could be rewritten as,

$$\Delta_{k+1} = {alph_k}/{NP\max_k} \tag{19}$$

where

$P\max_k = \max\{P_{k-N}, P_{k-N+1}, \ldots, P_k\} \approx \lambda_{\max}$ and $P_i = \mathbf{x}_i \mathbf{x}_i^T$.

D. The Narrow-Band Interferer

The three types of NBI considered were the single tone, dual tone and autoregressive interferers. The tone interferer consists of *m* complex sinusoids of the form,

$$i(k) = \sum_{l=1}^{m} \sqrt{P_l}\, e^{j(2\pi f_l k + \phi_l)} \tag{20}$$

where P_l and f_l are the power and normalised frequency of the lth sinusoid, and $\{\phi_l\}$ are independent random phases uniformly distributed on $(0, 2\pi)$. The autoregressive (AR) interferer, on the other hand, is modelled by a *p*th-order process given by,

$$i(k) = -\sum_{j=1}^{p} a_j i(k-j) + e(k) \tag{21}$$

where $e(k)$ is a white Gaussian process with variance ε^2.

III. SIMULATION RESULTS

The Gold-sequence of length 31 used as the spreading code, was generated using the polynomials $p^5 + p^2 + 1$ and $p^5 + p^4 + p^2 + p + 1$. The power levels of the tone interferers were set to $P_1 = P_2 = 10$ while the signal level of the second order AR(2) interferer was chosen to be 20 dB above the SS signal. The step-size for the LMS algorithm was chosen to be 0.0001 and the convergence factor for the RLS algorithm was set to 0.999 for all the results presented.

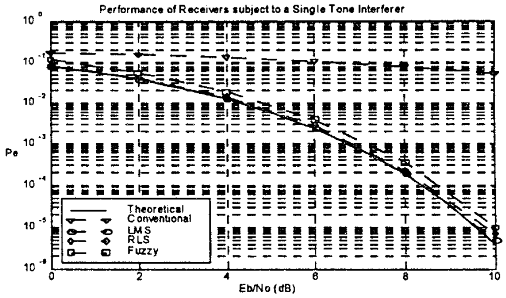

Fig. 6. Performance Curves for Signal Tone Interferer

Fig. 6 shows that the FLA method is marginally worse than the other techniques for a single tone interferer. This is also true for the dual tone interferer shown in Fig. 7.

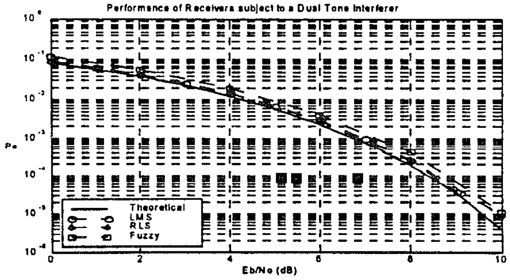

Fig. 7. Performance Curves for Dual Tone Interferer

It appears from Fig. 8 that the FLA performs better than the LMS algorithm for the chosen step-size. This performance advantage of the FLA can be directly attributed to its ability to change its step-size to accommodate the eigenvalue spread and variations for an AR(2) interferer. For Figures 6,7 and 8, an empirical choice of $\rho = 5$ was used in (18).

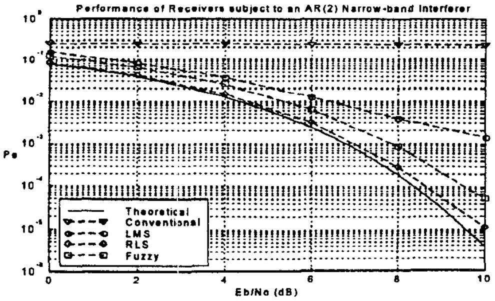

Fig. 8. Performance Curves for AR(2) Interferer

The RLS algorithm however, resulted in the best performance for the interference types considered. Unlike, the steepest gradient algorithm that can converge in N steps to the optimum coefficients, the RLS performs this in one step. This limitation is also present in the FLA technique since it also uses the steepest gradient algorithm to achieve its optimum coefficients.

On reviewing the *low* input membership function shown in Fig. 3, it can be seen that the FIS is forced to change the step-size to its maximum value for a zero MSE. This results in an increase in the residual MSE. The input function was then modified to reduce the membership of a zero MSE to *low*, has shown in Fig. 9 and Fig. 10 shows its corresponding FIS surface.

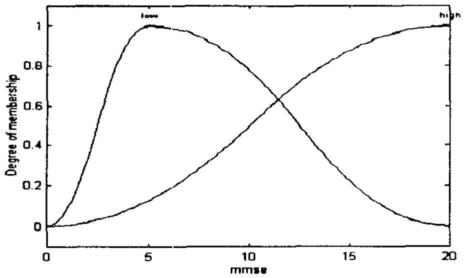

Fig. 9. Input Membership function of FIS *flmsc*

As was expected, the combination of modifying the input membership function and using (19), where a better estimate for the significant eigenvalue is use, a slight performance improvement was noticed for high signal to noise ratios, as shown in Fig. 11.

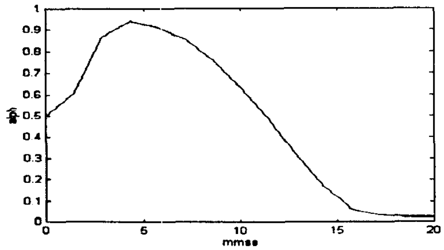

Fig. 10. *Output surface* for FIS *flmsc*

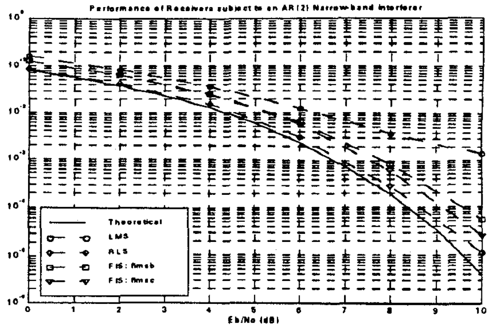

Fig. 11. FIS Performance Comparison

IV. CONCLUSIONS

We have considered in this paper the possible performance benefit of using a Fuzzy Logic Adaptation (FLA) technique for the detection of a SS signal in the presents of AWGN and NBI. Two possible fuzzy inference systems were proposed and compared in performance. It was further compared to the LMS and the RLS algorithms, where a performance improvement over the LMS algorithm has been show by simulation for an AR(2) interferer. The technique showed marginal degradation in performance for single and dual tone interferers.

The drawback of the FLA technique being based on the steepest gradient algorithm was explained, and the only way to compete with the performance of the RLS algorithm would be to reduce the number of interations required for convergence to the optimum steady state filter coefficients. This problems will be considered in future research.

Finally, this paper has demonstrated a method by which fuzzy logic could be used to suppress interference. Although its feasibility is shown by way of performance comparison with other schemes, a lot more work is required in this area of research.

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