PERFORMANCE EVALUATION OF A CODED CELLULAR SSMA SYSTEM

P.G.W. van Rooyen † and J.S. Kunicki ‡

† Alcatel Altech Telecoms, P.O. Box 286, Boksburg 1460, South Africa

‡ Cybernetics Lab., Rand Afrikaans University, P.O. Box 524, Johannesburg 2000

ABSTRACT

This paper investigates the performance and capacity of a cellular Direct Sequence Spread Spectrum Multiple Access (DS/SSMA) system. As modulation scheme coherent PSK and DPSK is assumed when the signal is corrupted by AWGN and inter user interference (IUI). Strong emphasis is placed on the performance and capacity improvements due to convolutional coding when hard decision decoding is employed. It is shown that the capacity of a SSMA system is very poor when an uncoded, non-cellular system is considered. When speech is transmitted in a coded cellular SSMA system it is shown that an increase of ten times is possible over existing TDMA systems.

1. INTRODUCTION

Spread spectrum signalling techniques, with its inherent anti- multipath, multiple access and rejection of interference capabilities, have increasingly received attention for cellular personal and mobile communications. Until recently the standard analysis of SSMA systems was rather pessimistic about the capacity of these systems compared to TDMA and even FDMA. Gilhousen et al [1] recognized that since SSMA capacity is only interference limited (unlike FDMA and TDMA which are primarily bandwidth limited), any reduction in interference converts directly and linearly into an increase in capacity. Therefore, by employing a voice activity factor, sectorizing the cells and using various forms of diversity it is possible to achieve SSMA system capacity at least as good as FDMA and TDMA.

Section 2 describes a cellular SSMA system that includes adjacent and reference cell interference. Also presented in this section is an analysis to determine the CPSK and DPSK performance.

Coded performance results will be presented in Sections 3 and 4 while Section 5 will conclude the paper by summarising the results and observations.

2. MODEL AND ANALYSIS

Using the equations derived in this work it is possible to predict DS/SSMA capacity under the mentioned conditions. We set out to predict system capacity and give some comparative results and show that simple coding can allow for reliable communication. By introducing a voice activity factor of 3/8 and cell splitting of 3, the performance of a 19 cell system with convolutional coding are assessed. We assume hard decisions are made by the demodulator and that the error-producing mechanism results in independent error events. The latter assumption requires interleaving at the transmitter and de-interleaving at the receiver.

2.1 Model

We assume that each user's code sequence has a period of $N = T/T_c$, where T is the data period and T_c the spreading sequence rate period. That is there is one period of spreading code sequence per data bit, and is generally known as the processing gain of a spread spectrum system. When coding is considered we assume that $N = T_s/T_c$, where $1/T_s$ is the coding rate. This is necessary since comparisons must be made on the basis of equal bandwidth. Further, when coding is used, the uncoded processing gain, N, must equal $\hat{N} = R_s N$ where R_s is the code rate.

The cellular layout is described by a uniform planar grid of J hexagonal cells with each cell containing a centrally located base station. The cells are divided into 120^o sectors, where each sector employs the same carrier frequency. The out stations are uniformly distributed throughout the system area with a density of K' users/cell, using DS/SSMA to establish a full- duplex channel with the base station. All cells are assumed to be equally loaded, with the up- and downlink transmission on two different frequencies.

Since the wideband SSMA signal is reused in every cell, the total interference at the cell site, to a given inbound station, is comprised of interference from other users in the same cell plus interference from users in neighbouring cells.

Our cellular system analysis differ from previous multiple cell analysis; a simple, yet effective, method is proposed to determine the cellular capacity. Measurements by Qualcomm [2] indicate that the first and second tier in a cellular system contribute approximately 6% and 0.2%, respectively, per cell of the total interference (for equally loaded cells). We therefore define a maximum interference parameter, K, that includes the interference from all users in all cells (adjacent plus reference cells). In other words, the maximum interference, K, can be written as

$$K = K' \frac{N_{sect}}{V_{on}} \{1.J1 + 0.06.J2 + 0.002.J3\}, \quad (1)$$

where K' is the total number of users/cell, V_{on} the voice activity factor, N_{sect} the cell splitting factor and $J2$ and $J3$ the number of interfering cells in the first and

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second tier surrounding the reference cell ($J1$). The third and higher tiers' interference is neglected.

For hexagonal cells $J2 = 6$ and $J3 = 12$ and therefore (1) can be written as

$$K = K' \frac{N_{sect}}{V_{on}} 1.4. \tag{2}$$

Using (2) we are in a position to analyze the **average** performance of a cellular SSMA system. In our performance calculations K will be calculated and K' (the number of users/cell) can then be determined. When only one cell without voice activity monitoring is considered, (1) simplifies to $K = K'$. However, it is evident that many of the inherent advantages of SSMA is lost when a non-cellular, non- voice system is considered.

2.2 Analysis

In our generic SSMA system, binary signalling is employed, hence the PSK waveforms are antipodal with probability of error, after matched filter reception, is given by

$$P_e = \frac{1}{2}\text{erfc}\left(\sqrt{\delta}\right) \tag{3}$$

for CPSK, and

$$P_e = \frac{1}{2}\exp(-\delta) \tag{4}$$

for DPSK and δ representing the signal-to-noise ratio of the received signal. The signal-to-noise ratio, corrupted by AWGN and inter user interference is given by [3, 4]

$$\delta = \left[2\sigma_{ma}^2 + \frac{N_0}{E_b}\right], \tag{5}$$

where σ_{ma}^2 is the multiple access variance and given by

$$\sigma_{ma}^2 = \frac{K-1}{3N}. \tag{6}$$

When the coded performance is considered we can use (3) and (4) together with the bounds described in equations (5.3.28), (5.3.29) and (5.3.33) in [5, pp. 302-303], for convolutional codes.

3. CPSK PERFORMANCE

We will start by investigating the uncoded performance of a cellular SSMA system, followed by evaluations of convolutional coding for similar conditions.

3.1 Uncoded CPSK Performance

We begin our investigation by looking at the influence of processing gain, N, on the capacity of a SSMA system with CPSK as modulation scheme. The average error rate is observed to increase with an increase in K and a decrease in N, which is as expected, and indicated in Figure 1, for a fixed multiple access interference, $K = 10$, and different values of N. We notice that, due to the finite cross correlation of the spreading sequences, the error rate saturates to an irreducible rate with increasingly unacceptable performance as N decreases. By increasing the processing gain from 15 dB ($N = 31$) to 21 dB ($N = 127$), a situation which is unacceptable for speech, can be improved dramatically. Also indicated in Figure 1 is the performance of ideal coherent PSK for reference purposes.

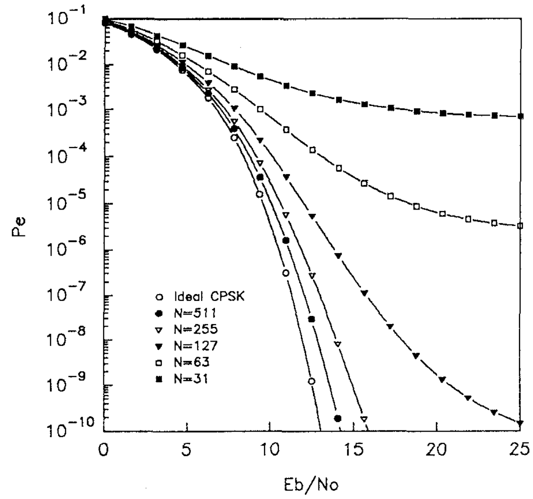

Figure 1: Uncoded CPSK performance, $K = 10$

Table 1 quantifies and tabulates K for different error rates (at $E_b/N_0 = 20$ dB). From this we see that K is exceedingly small for low values of N and P_e. Pickholtz et al [6] stated a rule of thumb for the capacity of a DS/SSMA system under AWGN conditions as being approximately 10% of the processing gain. At $P_e = 10^{-8}$ this would seem to be an accurate estimate.

An interesting point to note is that a change in processing gain results in a linear change in capacity. For example, by reducing the processing gain by 75% (i.e. from $N = 511$ to $N = 127$) the capacity is also reduced by 75%, in this case from 77 to 19 at 10^{-5}. This knowledge can be used quite productively when a capacity estimate is needed at a different processing gain, i.e. at $N = 1024$.

3.2 CPSK Convolutional Coding

The convolutional codes considered has standard generator polynomials, and are available from the literature, i.e. Table 5.3.1 [5, pp. 305] with code rates $R_{cd} = 1/2$, $1/4$ and $1/8$. These rates were chosen to make comparisons on an equitable base. For example, the performance of an uncoded, $N = 511$ system will be compared to a rate $R_{cd} = 1/2$, $N = 255$ system.

P_e	$N=511$	$N=255$	$N=127$	$N=63$	$N=31$
10^{-3}	152	76	38	19	9
10^{-5}	77	39	19	9	4
10^{-8}	41	21	10	5	2

Table 1: K for uncoded CPSK, $E_b/N_0 = 20$ dB

Both of these will result in the same data throughput and bandwidth.

The convolutional code constraint lengths are chosen to show how relatively simple codes ($\nu = 2, 3, 4$) perform. A rate $R = 1/2$, $\nu = 6$ encoder and decoder is commercially available and its performance is also presented.

Figure 2 indicates the average error performance as a function of the received signal-to-noise ratio for different coding rates, constraint length $\nu = 4$ and $K = 150$. The comparison is performed with approximately equal complexity. (Clark and Cain [7] indicates that convolutional codes with the same constrained length, irrespective of code rate has approximately equal complexity). The processing gain, N, of the coded signals is reduced by the code rate, R_{cd}, as explained earlier.

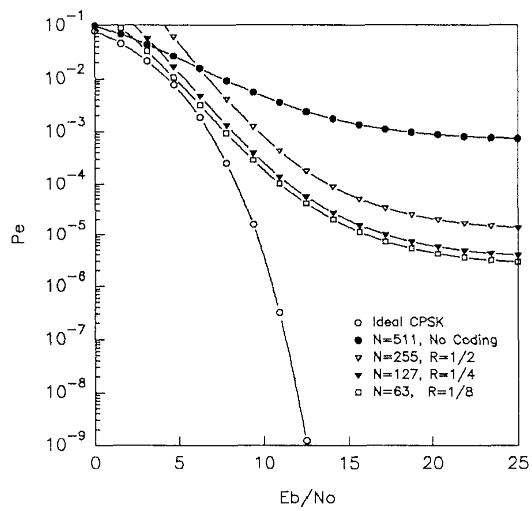

Figure 2: Convolutional coded CPSK performance, $\nu = 4$ and $K = 150$

We observe that low rate coding improves the average error rate considerably, and hence, will result in increased system capacity. An interesting point to note is that the rate $R_{cd} = 1/8$ code gives an error rate improvement at lower values of signal-to-noise ratio than higher rate codes when compared to the uncoded case.

Table 2 quantifies the convolutional coded capacity. We note that low rate coding increases the capacity significantly. At an average error rate of 10^{-5} a rate $R_{cd} = 1/8$, constrained length $\nu = 4$ convolutional code increases K by 108% over the uncoded case.

Of particular interest is the rate $R_{cd} = 1/2$, $\nu = 6$ code performance since, as mentioned, these coders are commercially available. Because of the code's long constrained length performance/capacity is improved over the low rate $R_{cd} = 1/8$, $\nu = 4$ code. Although it is not entirely fair to compare these codes to each other (different complexity) it is nevertheless instructive to know what is achievable in practice.

To highlight the main conclusion of this section; for **coherent PSK** low rate **convolutional** coding is very effective for capacity improvement. In the following section it will be shown that this is not necessarily the case when DPSK as modulation scheme is used.

4. DPSK PERFORMANCE

As with coherent PSK modulation, we first investigate the uncoded DPSK performance, followed by convolutional coded performance.

4.1 Uncoded DPSK

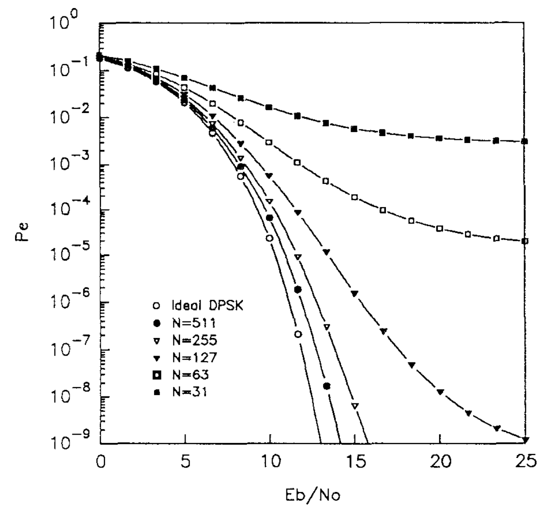

Figure 3: Uncoded DPSK performance, $K = 10$

Figure 3 depicts the average error rate as function of signal-to-noise ratio for $K = 10$ and different values of N. As in the CPSK case, we note the saturating error rate, with increasingly unacceptable performance as N decreases. Naturally with noncoherent detection one has inherently less gain available, resulting in higher saturation of error rates. By comparison, for $N = 63$, DPSK signalling saturates at an average error rate of almost an order of magnitude higher than CPSK.

Table 3 tabulates the capacity of an uncoded DPSK system for different values of average error rate ($E_b/N_0 = 20$ dB). Comparing Tables 3 and 1 we notice that at low error rates, i.e. 10^{-8}, the capacity for DPSK signalling is not significantly less than for

	$N=511$ No Coding	$N=255, R_{cd}=1/2$				$N=127, R_{cd}=1/4$			$N=63, R_{cd}=1/8$		
		ν				ν			ν		
P_e		2	3	4	6	2	3	4	2	3	4
10^{-3}	152	190	208	226	263	183	228	256	203	226	270
10^{-5}	77	113	123	141	170	106	130	156	116	134	160
10^{-8}	41	66	69	86	110	60	75	95	68	80	98

Table 2: K for convolutional coded CPSK, $E_b/N_0 = 20$ dB

P_e	$N=511$	$N=255$	$N=127$	$N=63$	$N=31$
10^{-3}	115	57	28	14	6
10^{-5}	63	32	16	8	4
10^{-8}	36	18	9	4	2

Table 3: K for uncoded DPSK signalling, E_b/N_0

CPSK. However, at high error rates, i.e. 10^{-3}, the capacity difference is in the order of 24% less with DPSK signalling.

It is thus evident that there is an interesting tradeoff between CPSK and DPSK. Since systems are normally designed to operate at low error rates DPSK as modulation scheme, due to its lower complexity, must definitely be considered. However, as mentioned earlier, the uncoded capacity of SSMA is very low when compared to other multiple access techniques. In the concluding section of this chapter we will show that a multiple cell architecture can allow for acceptable uncoded SSMA performance, be that CPSK or DPSK.

4.2 DPSK Convolutional Coding

The same convolutional codes as used in the CPSK case are considered, again under equal complexity and throughput.

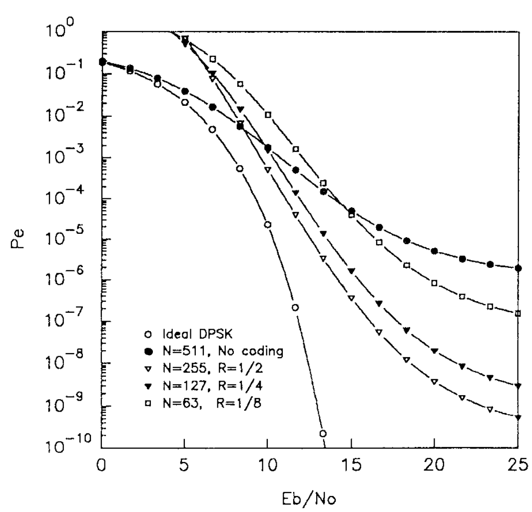

Figure 4: Convolutional coded DPSK performance, $\nu = 4$ and $K = 60$

This behaviour is significantly different from that observed with CPSK because of the noncoherent combining loss which increases with the code free distance. A coherent modulator is able to utilise the additional code free distance more efficiently, and these characteristics is not observed. These observations are also consistent with non-spread spectrum systems [7].

Indicated in Figure 4 is the error rate performance as a function of received signal-to-noise ratio for different coding rates, constraint length $\nu = 4$ and $K = 60$. It is clear that low rate coding degrades the performance relative to the uncoded and half rate coded cases.

Table 4 quantifies the convolutional coded performance. At 10^{-5} the commercially available, half rate, $\nu = 6$ code delivers a capacity increase of 75%, while the lower rate codes deliver a constantly worse performance as the code rate is lowered.

Comparing Tables 4 and 2 we recognise that coding accentuates the small capacity difference present in the uncoded case. For the half rate, $\nu = 6$ code the DPSK capacity is 36% worse than the corresponding CPSK case at 10^{-5}.

It is interesting to note that the linearity observed for the uncoded case is maintained under coded conditions. For example, from Tables 2 and 4, changing N by 50% (i.e. $N = 127$ to $N = 63$) the capacity is also changed by 50%.

5. CONCLUSIONS AND DISCUSSION

Results have been presented to determine the capacity of a SSMA system under AWGN conditions. The results indicate that a SSMA system without coding is not very bandwidth efficient. The coding results verify, and to a large extent quantify, articles written mainly by Viterbi on the subject of combined coding and SSMA [8, 9]. In these articles Viterbi makes the claim that very low rate coding and spread spectrum signalling provides optimum system capacity. We have shown that with coherent PSK with convolutional coding this seems to be indeed correct. However, when noncoherent detection is used this statement is incorrect. We will now show that by improved system architecture the SSMA capacity can be further improved.

As mentioned earlier, K is the maximum interference that the cellular system can tolerate. For the moment let us consider one cell with no cell splitting or voice activity factor. In other words all the tables and figures presented is valid for a one cell system. Since no voice

	$N = 511$ No Coding	$N = 255, R_{cd} = 1/2$				$N = 127, R_{cd} = 1/4$			$N = 63, R_{cd} = 1/8$		
		ν				ν			ν		
P_e		2	3	4	6	2	3	4	2	3	4
10^{-3}	115	120	128	135	152	95	112	122	81	87	98
10^{-5}	63	78	84	94	109	62	74	85	54	60	69
10^{-8}	36	49	52	62	75	39	47	57	36	41	48

Table 4: K for convolutional coded DPSK, $E_b/N_0 = 20$ dB

activity factor is included K also signifies the capacity for a data communication channel.

As a specific example, if data is transmitted at 64 kbps and a moderate length spreading sequence is used with CPSK as modulation scheme, say $N = 127$, our system can support only 19 channels (Table 1) at $P_e = 10^{-5}$ with an RF bandwidth in excess of 16 MHz. If a half rate, constrained length $\nu = 6$ convolutional code is used the capacity is increased by 115% to 41 channels. Even when coding is considered, it is quite clear that the system capacity is relatively low.

Considering a 19 cell system (18 interfering cells), (2) can be written as $K' = \frac{3K}{1.4}$ users/cell which includes cell splitting. (Since we are considering data transmission for the moment the voice activity factor V_{on} is neglected.) It is clear that a cellular architecture more than doubles the SSMA capacity. Therefore, coding together with a cellular architecture can increase data communication system capacity by almost 360% - a substantial increase.

However, our biggest increase is obtained when speech is transmitted. Consider again a 19 cell system (18 interfering cells), (2) can be written as $K' = \frac{8K}{1.4}$ users/cell which includes voice activity monitoring and cell splitting. We notice that a capacity increase of five times can be gained by an enhanced system architecture. Returning to our previous example ($N = 127$ with coding), instead of 19 users our system can now support 234 users/cell - a massive increase of 1131%! It must be stressed that TDMA system capacity cannot be increased in this manner - this is a unique feature of SSMA.

By comparison, the Rurtel TDMA system, backed by Alcatel Altech Telecoms, supports twenty-eight 64 kbps channels in a 4 MHz bandwidth, translating to 28 users/cell.

To determine the bandwidth efficiency of the mentioned TDMA system and the analyzed SSMA system we can determine the users/MHz as an indication of the bandwidth efficiency. For the CPSK SSMA system with half rate, $\nu = 6$ convolutional coding at 10^{-5} error rate, 971 users at 64 kbps use a bandwidth of 32.7 MHz (Table 2 for $N = 255$). This translates to 30 users/MHz. For DPSK modulation under similar conditions as the CPSK case we have 623 users and 19 users/MHz (36% less than for CPSK).

To determine the bandwidth efficiency of the Rurtel system in a cellular setup we can argue as follows. It is well known that the frequency reuse factor of TDMA cellular is seven, meaning that the same frequency can only be reused every seventh cell. Including guard bands, which is typically 6 MHz, the Rurtel TDMA system uses a bandwidth of six times 6 MHz (guard bands) plus seven times 4 MHz, equalling a total bandwidth of 64 MHz with seven times 28 channels. This results in 3 users/MHz - 10 times less than CPSK and almost six times less than DPSK.

It must be stressed that coding and voice activity monitoring can not be utilised as efficiently in TDMA systems than in SSMA systems.

The additional advantage to be gained from a SSMA system is that virtually a whole country can be covered on one frequency, provided a large enough family of spreading codes is available.

A typical application of these results will be to predict the performance and/or capacity of a stationary cellular system, of which a stationary IWC network (under certain conditions), or a rural telephone system are typical examples.

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