

ON THE CONSTRUCTION OF LAYERED SPACE-TIME CODED MODULATION (STCM) CODES EMPLOYING MTCM CODE DESIGN TECHNIQUES

D.J. VAN WYK¹, I.J. OPPERMAN², E. PRETORIUS³ AND P.G.W. VAN ROOYEN⁴

¹ DEFENCETEK, CSIR, Pretoria, South Africa (dvwyk@csir.co.za)

² CWC, University of Oulu, Oulu, Finland (ian.oppermann@ee.oulu.fi)

³ Reutech Defence Industries, South Africa (etiennep@rdi.co.za)

⁴ EEE, University of Pretoria, Pretoria, South Africa (pvrooyen@postino.up.ac.za)

Abstract: The field of Space-Time Coded Modulation (STCM) has recently attracted interest as a means of improving link reliability in mobile environments. It has been shown that these space-time multiple-antenna systems provide very high capacity when compared to single antenna systems in a Rayleigh fading environment. Specifically for the forward-link, space-time coded transmit diversity should be incorporated into the wireless communication system in order to take full advantage of the combined space- (multiple-transmit antennas) and time- (forward error correction coding) diversity scheme. In this paper, we adopt a heuristic approach to the design and evaluation of diagonally layered STCM schemes by utilising classical Multiple-Trellis Coded-Modulation (MTCM) techniques. We will focus mainly on the construction of simple 4-state diagonally layered space-time convolutionally coded systems. The designed STCM systems will be used in a comparative performance investigation with the originally proposed STCM codes. Computer simulations are used to evaluate the performance of the code designs on flat Rayleigh fading channels.

I. INTRODUCTION AND BACKGROUND

An important new area of diversity combining is space-time processing. Previous works [1, 2] have established the importance of spatial diversity and the impact on the capacity of wireless communication systems. The latter has focused mainly on the reverse link channel (mobile-to-base). Bearing in mind the strict complexity requirements of handsets, and the characteristics of the forward link channel, a brute-force application of advanced detectors with multiple receiving antennas is not seen as the desired solution to the forward link capacity problem. Alternative solutions have been proposed suggesting that multiple transmit antennas (or transmit diversity) at the base station will increase forward link capacity with only minor increase in mobile handset implementation.

In this paper we focus on the construction of diagonally layered space-time convolutional codes to TDMA systems. Specifically, a TDMA system similar, but not identical, to the IS-136 US cellular standard is considered. The gain from space time coding comes from the antenna diversity as well as the use of coding between signals transmitted in parallel from antennas. The symbols from the coded information words are transmitted simultaneously (applying a block orthogonalisation scheme), or with known time offsets, from the n_{tx} possible transmit antenna elements. A code symbol is now defined by an antenna element

and a symbol constellation point. A code sequence is defined by a series of code symbols constraint by current data and previous inputs in a convolutional coding manner. By maximizing the distance between the code symbols, the technique can achieve a coding gain in a manner similar to conventional coding schemes.

Recall, the appropriate criterion for designing good TCM schemes for the AWGN channel is to maximise the minimum Euclidean Distance (ED) between any two distinct information sequences of the coded sequences. Several papers [3, 4] have showed that the error rate performance of Trellis-Coded Modulation (TCM) schemes over fading channels can be strongly influenced by the effective or shortest Error Event Path (EEP), L_{min} and the minimum product distance, λ_L along that error event path. These parameters play a more important role than the minimum ED. For this reason Multiple Trellis-Coded Modulation (MTCM) codes have been designed in order to achieve superior performance on the fading channel, compared to that achievable by conventional Trellis-Coded Modulation (TCM) of the same throughput and decoder complexity.

In this paper, we adopt a heuristic approach to the design and evaluation of diagonally layered STCM schemes by employing classical (MTCM) techniques. The advantage of the latter approach is that it provides an unified design procedure for these STCM systems, including the design of multi-level, multi-dimensional and asymmetric coded modulation schemes. In [5], Tarokh *et al* proposed two design criteria, namely the "Rank" and "Determinant" criteria, for the design of STCM systems. In addition they have presented two design rules for the construction of these codes assuming a phase-modulated system. We will introduce design criteria, more closely related to the MTCM code construction process, to design these STCM systems. Using this insight, we will define the so-called Layered Squared Euclidean Distance (LSED) and Layered Squared Euclidean Distance Product (LSEDP) of the STCM system. The applicability of the latter MTCM design approach will be evaluated against the design criteria proposed in [5].

The paper is organised as follows: The general system model for multiple antenna systems and space-time codes is revisited in Section 2. We introduce the MTCM code design techniques and a framework for evaluation the effectiveness of the designed coded systems. In Section 3, we present the design of 4-state convolutionally encoded STCM systems. In Section 4 the Bit Error Rate (BER) performance evaluation of the designed STCM systems under flat Rayleigh fading channel conditions is considered. A comparative performance evaluation

between codes designed with the proposed MTCM design approach and the originally proposed design criteria for STCM codes is carried out. In Section 5, we present a discussion on the effectiveness and penalties associated with employing the proposed MTCM based design criteria.

II. SPACE-TIME CODED SYSTEM MODEL

The block diagram of the space-time coded system under investigation is shown in Fig. 1. The input binary stream is first passed through the space-time encoder and signal-point mapper. After mapping successive blocks of b bits into the M -ary PSK constellation, the encoded symbols are passed through a block symbol interleaver.

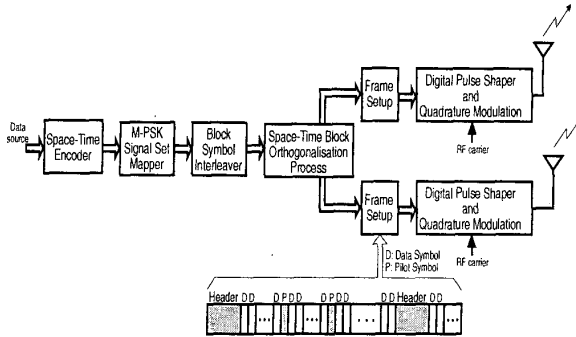


Fig. 1. Transmitter block diagram of space-time coded system.

In the space-time block orthogonalisation module the symbols are encoded to maintain orthogonality (full-rank) and multiplexed into the n_{tx} -antenna streams. Specifically, following Alamouti's scheme [6], this encoding and multiplexing for $n_{tx} = 2$ results in the following transmit diversity sequence (T_s is the symbol duration):

	antenna 0	antenna 1
time t	s_0	s_1
time $t + T_s$	$-s_1^*$	s_0^*

In addition, the known reference headers and pilot symbols are inserted. The position of the pilot symbols within the frame has an insignificant effect on bit error performance [7]. The reference header of length L_H symbols (known to the receiver) is utilised to establish synchronisation (time, clock, and symbol), and also to indicate the state of the fading during the header transmission. In addition to this a known pilot symbol is inserted in every frame of length L_N symbols. This frame transmission strategy is also illustrated in Fig. 1. After multiplexing, header/pilot symbol insertion, the I (in-phase) and Q (quadrature-phase) baseband signals are modulated onto a carrier of frequency f_c and transmitted over the n_{tx} antennas.

For personal communication systems in a micro-cellular radio system, the fading effects can be modeled as being non-frequency selective (or flat fading), with a Rayleigh amplitude and uniform phase distribution. These channels are characterized by the Rician parameter, K , and the product of the maximum Doppler frequency, f_D , and the symbol duration, T_s . We denote the distortion on the antenna paths as $h_0(t)$ and $h_1(t)$. We follow Foschini and Gans [8] in assuming that the fading is flat fading and quasi-static; that is constant over a frame and

varying from one to the other. Thus, the fading is constant across two consecutive symbols under these (quasi-static) conditions and we have $h_i(t) = h_i(t + T_s) = h_i$, $i = 1, 2$.

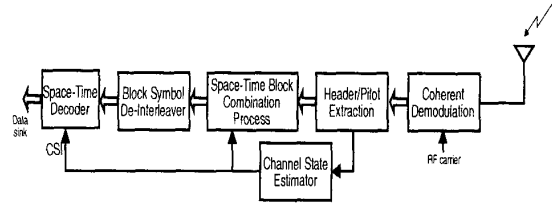


Fig. 2. Space-time coded system receiver block diagram.

Figure 2 shows the structure of the space-time coded receiver where it is assumed that only a single antenna is utilised at the receiver. The received signal is coherently demodulated with a locally generated carrier reference, assuming perfect carrier and time synchronisation. The demodulated and low-pass filtered complex signals, entering the space-time block combination module, are given by

$$r_0 = r(t) = h_0 s_0 + h_1 s_1 + \eta_0 \quad (1)$$

$$r_1 = r(t + T_s) = -h_0 s_1^* + h_1 s_0^* + \eta_1 \quad (2)$$

In the above, η_0 and η_1 are complex random variables as a result of receiver noise and interference, modeled as independent zero mean white Gaussian noise processes.

The space-time block combination module, performs a de-orthogonalisation procedure, resulting in the combined signals, \tilde{s}_i , expressed as

$$\tilde{s}_0 = h_0^* r_0 + h_1 r_1^* \quad (3)$$

$$\tilde{s}_1 = h_1^* r_0 - h_0 r_1^* \quad (4)$$

In the case of perfect channel estimation, the Channel State Information (CSI) vector, described in terms of the channel response vectors $h_i(t)$, is completely known at the receiver. In practice, the inserted header symbols are utilised in the channel estimator to derive an estimate of the fading amplitude and phase over the header interval. This estimate is used as non-perfect CSI in the space-time decoder, and is constantly updated by extraction of the fade estimate.

It is important to note that even if perfect fading estimation (ideal CSI) is available, the channel can never be compensated to a degree to achieve AWGN-like performance. This is because the demodulation system with CSI is carried out with regard to the received signal, which consists of the faded original signal plus an additional noise component.

During the decoding process, the space-time decoder takes into account the state of the fading channel by means of CSI based metric weighting functions. This scheme assumes the channel to be unreliable when deeply faded and gives lower weights to branch metrics computed during such intervals. After decoding and decision making the decoded bit stream is recovered and delivered to the output.

III. SPACE-TIME CODE DESIGN

In its most general form, MTCM is implemented by an encoder with b binary input bits and s binary output bits that are mapped

into k M -PSK symbols in each transmission interval. The parameter k is referred to as the *multiplicity* of the code, since it represents the number of M -PSK symbols allocated to each branch in the trellis diagram ($k = 1$ corresponds to conventional TCM). To produce such a result, the s binary encoder output bits are partitioned into k groups containing m_1, m_2, \dots, m_k symbols. Each of these groups, through a suitable mapping function, results in a M -PSK output symbol.

Recall that with conventional trellis coding (i.e., one symbol per trellis branch), the length L_{min} of the shortest EEP is equal to the number of trellis branches along that path. A trellis diagram with parallel paths is constrained to have a shortest EEP of one branch, $L_{min} = 1$. This implies that the asymptotic region of the graph of average bit error probability will vary inverse linearly with \overline{E}_s/N_o or E_s/N_o , since $\overline{E}_s = E_s$ [9]. Therefore, from an error probability viewpoint it is undesirable to design conventional TCM codes to have parallel paths in their trellis diagrams.

When the MTCM approach is employed for space-time code designs, the option of designing a trellis diagram with parallel paths may again be considered, since it offers more flexibility in selecting higher effective code lengths (or error event paths). The reason behind this lies in the fact that even if parallel paths exist in the trellis, it is now possible to have more than one coded symbol with non-zero ED associated with an EEP branch of length, $L_{min} = 1$.

In the design of the space-time codes a procedure similar to that presented in [9], known as the *Ungerboeck: From Root-to-Leaf* approach, has been followed. The set partitioning method, makes use of k -fold Cartesian products of the sets found in Ungerboeck's original set-partitioning method for conventional trellis codes [10]. The set-partitioning procedure is started with a k -fold Cartesian product of the complete M -PSK signal set.

The latter multiplicity factor is the most important parameter in the space-time coding design procedure. In general, the design criteria do not include any direct considerations on the choice of multiplicity factor, k , as a function of the channel parameters. In [11], we have shown how the general design criteria for MTCM codes for fading channels, can be augmented by including an analysis of the lengths of burst errors. For the space-time transmission system under consideration the choice of k is naturally determined by the number of available transmit antennas, n_{tx} .

A. Ungerboeck Set Partitioning:

Considering the code design for $n_{tx} = 2$, the first step is to partition $A_0 \otimes A_0$ into M signal sets defined by the ordered Cartesian product $\{A_0 \otimes B_i\}$, $i = 0, 1, \dots, M - 1$. The second element $\{j_2\}$ of B_i is defined by $n_j + i \text{ mod } M$. In terms of the space-time mapping it is appropriate to define a new design parameter, which will be called the Layered Squared Euclidean Distance (LSED). Specifically, since the LSED between any pair of $k = 2$ -tuples is the sum of the distances between corresponding symbols in the 2-tuples, the set partitioning guarantees that the *intradistance* (i.e., distance between pairs within a specific set or partition) of all of the partitions $A_0 \otimes B_i$ is identical. In addition, as a result of the possible existence of parallel paths in the decoding trellis, the minimum product of LSEDs must be maximized. We refer to this parameter as the LSEDP, which is given by $\prod d_{ij}^2$.

Therefore, for the generating set $A_0 \otimes B_0$, the minimum

LSEDP over all pairs of 2-tuples must be maximized. This is done by choosing the odd integer multiplier, n such that it produces the desired *maximin* solution. A computer search for possible values of n , revealed the solution to be $n = 1$. The sets, $A_0 \otimes B_i$, $i = 1, \dots, M$ for $n_{tx} = k = 2$, are illustrated below for M -PSK with $M = 4$ (QPSK).

$$\begin{aligned} A_0 \otimes B_0 &= \begin{bmatrix} 0 & 0 & 1 & 1 & 2 & 2 & 3 & 3 \end{bmatrix} \\ A_0 \otimes B_1 &= \begin{bmatrix} 0 & 1 & 1 & 2 & 2 & 3 & 3 & 0 \end{bmatrix} \\ A_0 \otimes B_2 &= \begin{bmatrix} 0 & 2 & 1 & 3 & 2 & 0 & 3 & 1 \end{bmatrix} \\ A_0 \otimes B_3 &= \begin{bmatrix} 0 & 3 & 1 & 0 & 2 & 1 & 3 & 2 \end{bmatrix} \end{aligned}$$

Note that each set has a minimum intradistance of $4E_b$. The *interdistances* (i.e., minimum distances between pairs of 2-tuples from different sets), for these sets are summarised in Table 1.

Table 1. *Interdistances between partitioned subsets, with $A_0 \otimes B_0$ used as reference. $M = 4$.*

Subset	Distance	Subset	Distance
$A_0 \otimes B_0$	—	$A_0 \otimes B_1$	$4E_b$
$A_0 \otimes B_2$	$8E_b$	$A_0 \otimes B_3$	$4E_b$

Following tradition, the following steps in the set-partitioning procedure are to partition each of the M sets $A_0 \otimes B_i$, into two sets $C_0 \otimes D_{i0}$ and $C_0 \otimes D_{i1}$, with the first containing the even elements ($j = 0, 2, \dots, M - 2$) and the other containing the odd elements ($j = 1, 3, \dots, M - 1$).

The sets, $C_0 \otimes D_{ij}$, using the procedure described in the foregoing are illustrated below.

$$\begin{aligned} C_0 \otimes D_{00} &= C_0 \otimes D_{20} = \begin{bmatrix} 0 & 0 & 2 & 2 \end{bmatrix} \\ C_0 \otimes D_{01} &= C_0 \otimes D_{21} = \begin{bmatrix} 1 & 1 & 3 & 3 \end{bmatrix} \\ C_0 \otimes D_{10} &= C_0 \otimes D_{30} = \begin{bmatrix} 0 & 2 & 2 & 0 \end{bmatrix} \\ C_0 \otimes D_{11} &= C_0 \otimes D_{31} = \begin{bmatrix} 1 & 3 & 3 & 1 \end{bmatrix} \end{aligned}$$

Note that each set has a minimum intradistance of $8E_b$ and the interdistances for these sets are $8E_b$.

IV. ENCODER AND DECODER IMPLEMENTATION

A. Encoder Analytical code description:

The Calderbank—Mazo analytic description of the trellis codes was developed for one dimensional modulation, and channel signals have to be real numbers for the analytic process. The output channel signals are directly expressed in terms of a sliding block of input bits, with the intermediate step of output coded bits being irrelevant for analytical described trellis codes [9, 12].

When a trellis code is used to encode data at a rate of r bit per channel symbol, each channel input x will depend not only on the most recent block of k bits that enter the encoder but will also depend on the ν bits preceding this block. The output symbol may be determined by solving the set of equations resulting from all the input bit combinations

$$\begin{aligned} &x(b_1, b_2, \dots, b_n) \\ &= \sum_{i=1}^n d_i b_i + \sum_{i,j=1; j>i}^n d_{ij} b_i b_j \\ &+ \sum_{i,j,l=1; l>j>i}^n d_{ijl} b_i b_j b_l + \dots + d_{12\dots n} b_1 b_2 \dots b_n \end{aligned}$$

or by solving its equivalent matrix equation, describing the encoder function as follows

$$X = B \cdot D \quad (5)$$

where $b_i = 1 - 2a_i$, X is the channel signal matrix, B is the Hadamard matrix with elements ± 1 and D the matrix of constants which determines the code. In order to calculate the coefficients of matrix D , the matrix of the channel symbols is calculated from the trellis diagram and the solution for D is obtained from

$$D = \frac{B^T X}{2^n} \quad (6)$$

where X is the channel signal matrix, B is the Hadamard matrix with elements ± 1 , and D is the matrix of constants to be determined.

B. Design of 4-State Space-Time codes:

In this sub-section a rate-2/4 space-time trellis code of multiplicity $k = 2$, is designed. Two QPSK symbols are transmitted over the channel for every 2 bits accepted by the encoder. The input/output/state connection diagram for this coding system is shown in Fig. 3. It defines the sliding block of source variables $(b_1, b_2, b_3^{(1)}, b_4^{(1)}, b_3^{(2)}, b_4^{(2)})$. Note that the output bits $(b_3^{(k)}, b_4^{(k)})$, $k = 1, 2$ are mapped into the QPSK symbols output from the antenna elements.

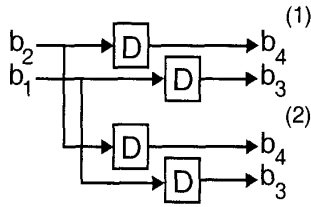


Fig. 3. Space-Time encoder input/output/state connection diagram.

The code structure for the half-connected rate-2/4 space-time code is presented in Fig. 4, for a cardinality of 2. The number of branches associated with each state (i.e., emanating from or terminating in a node) equals $2^2 = 4$.

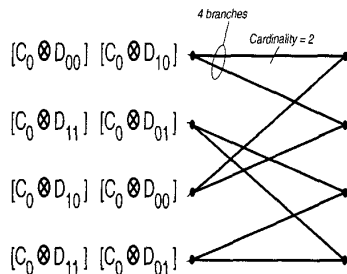


Fig. 4. Trellis diagram of 4-state space-time code ($n_{tx} = 2$).

Substituting the coder output code words for the M -PSK code words, and by using the Calderbank—Mazo algorithm, the

following solutions for the $D^{(k)}$, $k = 1, 2$ matrices are obtained

$$\begin{aligned} d_1^{(1)} &= 2, d_{13}^{(1)} = -1, \\ d_{124}^{(2)} &= -1, d_{1234}^{(2)} = 2 \end{aligned} \quad (7)$$

Substituting (7) into (5) for $n = 4$, the analytical description of the encoder is given by

$$\begin{aligned} X^{(1)} &= 2b_1 - b_{13} \\ X^{(2)} &= -b_{124} + 2b_{1234} \end{aligned} \quad (8)$$

where $X^{(k)}$ denotes the QPSK symbol output from the $(k-1)$ th antenna.

In the following section, the performance of this code will be benchmarked against the rate-2/4 space-time trellis code presented by Tarokh [5] (For the trellis diagram the reader is referred to Fig. 4 of this paper). Following the procedures described in the foregoing, the analytical description of this space-time encoder is given by

$$\begin{aligned} X^{(1)} &= 2b_4 - b_{34} \\ X^{(2)} &= 2b_2 - b_{12} \end{aligned} \quad (9)$$

Note that the analytical representation of the trellis encoders encapsulates the combined modulation/coding process without the necessity of separating it into its component parts.

V. PERFORMANCE EVALUATION

This section presents computer simulation results of the uncoded and coded QPSK communication system, operating at 2.0 bits/s/Hz. For all the simulations, a frame length N of 200 symbols, header length L_H of 8 symbols, and sub-frame length L_N of 8 symbols were assumed. N was also taken as the decoding depth of the Viterbi decoder. The effect of varying N and L_N is left for further study. A vehicle speed of 120 km/h, and a carrier frequency of 2 GHz were assumed.

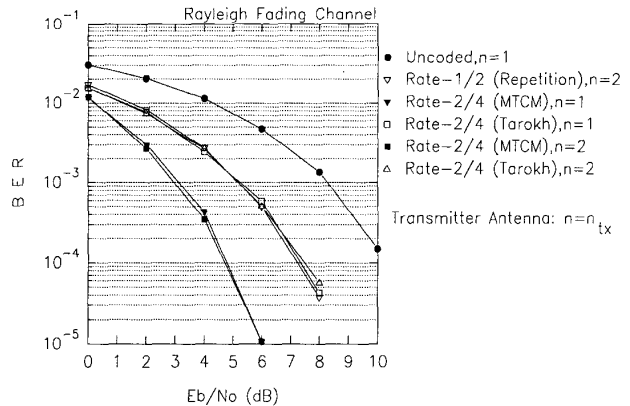


Fig. 5. QPSK BER results for uncoded and 4-state rate-2/4 space-time coded QPSK system operating on the AWGN channel, when $n_{tx} = 1, 2$ antennas are used at the transmitter.

Fig. 5 and 6 depict the BER results of the rate-2/4 space-time coded QPSK systems, compared to the uncoded system performance under AWGN and fast Rayleigh fading conditions, respectively. For these performance curves transmit diversity

order of $n_{tx} = 1$ and 2 have been assumed. In addition, the BER performance results of a QPSK system employing 2 transmit antennas and rate-1/2 repetition coding are also shown. For all the coded systems, a block symbol interleaver (pseudo-random) of size N has been utilised.

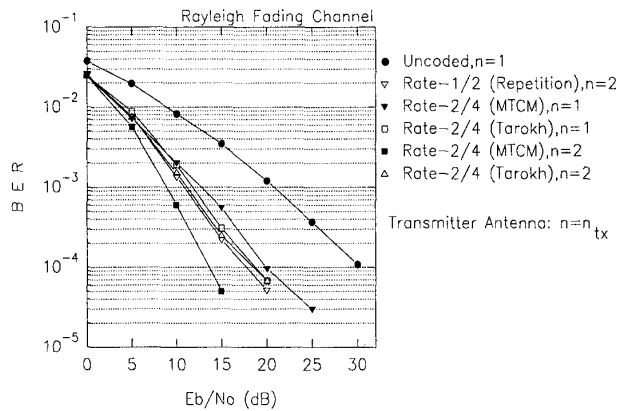


Fig. 6. QPSK BER results for uncoded and 4-state rate-2/4 space-time coded QPSK system operating on the flat fading Rayleigh channel, when $n_{tx} = 1, 2$ antennas are used at the transmitter.

From these figures, the newly designed (MTCM based) space-time coded systems ($n_{tx} = 2$) outperformed both the repetition code and the previously proposed space-time codes. It can be seen that for both the AWGN and Rayleigh channels coding gains, relative to both the uncoded and repetition coded systems, are obtained for the MTCM-based space-time codes.

It is interesting to note that the originally proposed 4-state space-time code fails to significantly improve on the performance of rate-1/2 repetition code for both channels considered. The latter can be attributed to the fact that the latter codes were designed by hand to maximise the coding advantage given by the determinant criterion. In this instance it is clear that the determinant criterion is approximate, since it does not take in account the path multiplicity.

The BER performance results for the Rayleigh fading channel are summarized in Table 2.

Table 2. Performance comparison of the uncoded and space-time coded QPSK system, operating on the flat fading Rayleigh channels at a BER of $P_b = 10^{-4}$ dB.

Designed	Rate rate	Code	E_b/N_o for $P_b < 10^{-4}$	Coding Gain ⁽¹⁾
MTCM	2/4	$n_{tx} = 1$	20 dB	11 dB
Tarokh	2/4	$n_{tx} = 1$	19 dB	12 dB
Repetition	1/2	$n_{tx} = 2$	18 dB	13 dB
MTCM	2/4	$n_{tx} = 2$	13.5 dB	16.5 dB
Tarokh	2/4	$n_{tx} = 2$	18.5 dB	12.5 dB

(1) Gain over uncoded QPSK with $n_{tx} = 1$.

VI. CONCLUSIONS

The MTCM design criteria presented in the open literature [3, 4, 9] were utilised in the designs of new space-time trellis codes for M -PSK systems. In particular, to minimise the error

probability, the code has to maximise LSEDP of symbols at non-zero ED from the shortest EEP. The secondary objective is to maximise the LSEDP of the branch distances along the shortest EEP. The final objective then, is to maximise the free squared ED of the code. This study did not render the design criteria presented in [13] obsolete. In fact, there are many similarities when comparing the originally proposed rank criteria, with the LSEDP criteria, and the determinant criteria with the presented LSEDP criteria. Rather, this work has highlighted the existence of design tools (readily available and well-understood) which can be used to assist in the design and evaluation of future space-time coded systems.

REFERENCES

- [1] J. H. Winters, J. Salz, and R. D. Gitlin, "The impact of antenna diversity on the capacity of wireless communications systems," *IEEE Transactions on Communications*, vol. 42, pp. 1740-1751, February/March/April 1994.
- [2] A. J. Paulraj and E. Lindskog, "Taxonomy of space-time processing for wireless networks," *IEE Proceedings on Radar, Sonar and Navigation*, vol. 145, pp. 25-31, February 1998.
- [3] D. Divsalar and M. K. Simon, "The design of Trellis Coded MPSK for fading channels: Performance Criteria," *IEEE Transactions on Communications*, vol. 36, pp. 1004-1012, September 1988.
- [4] D. Divsalar and M. K. Simon, "The design of Trellis Coded MPSK for fading channels: Set Partitioning for optimum code design," *IEEE Transactions on Communications*, vol. 36, pp. 1013-1021, September 1988.
- [5] V. Tarokh and N. S. A. R. Calderbank, "Space-time codes for high rate wireless communication: Performance criterion and code construction," *IEEE Transactions on Information Theory*, vol. 44, pp. 744-765, March 1998.
- [6] S. M. Alamouti, "A simple transmit diversity technique for wireless communications," *IEEE Journal on Selected Areas of Communications*, vol. 16, pp. 744-765, October 1998.
- [7] J. K. Cavers, "An analysis of pilot symbol assisted modulation for Rayleigh fading channels," *IEEE Transactions on Vehicular Technology*, vol. 40, pp. 686-693, November 1991.
- [8] G. J. Foschini and M. J. Gans, "On the limits of wireless communication in a fading environment when using multiple antennas," *Wireless Personal Communication*, vol. 6, pp. 311-335, March 1998.
- [9] E. Biglieri, D. Divsalar, P. J. McLane, and M. K. Simon, *Introduction to Trellis-Coded Modulation with Applications*. Macmillan, 1991.
- [10] G. Ungerboeck, "Channel coding with Multilevel/Phase signals," *IEEE Transactions on Information Theory*, vol. IT-28, pp. 55-67, January 1982.
- [11] D. J. van Wyk, M. P. Lötter, L. P. Linde, and P. G. W. van Rooyen, "A multiple trellis coded Q²PSK system for wireless local loop," in *PIMRC'97: International Symposium on Personal Indoor and Mobile Radio Communications*, (Helsinki, Finland), pp. 624-628, September 1997.
- [12] R. Calderbank and J. E. Mazo, "A new description of trellis codes," *IEEE Transactions on Information Theory*, vol. IT-30, pp. 784-791, November 1984.
- [13] V. Tarokh, A. Naguib, N. Seshadri, and A. R. Calderbank, "Low-rate multi-dimensional space-time codes for both slow and rapid fading channels," in *PIMRC'97: International Symposium on Personal Indoor and Mobile Radio Communications*, (Helsinki, Finland), pp. 1206-1210, September 1997.