

Performance of DS-CDMA systems with Antenna Arrays in Non-Uniform Propagation Environments

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Abstract— In this paper, the Bit Error Rate (BER) of a Direct Sequence Code Division Multiple Access (DS-CDMA) cellular system incorporating antenna arrays for spatial filtering is determined when operated in an environment where the propagation characteristics is a function of the Direction of Arrival (DOA) of signals. Specifically, this paper analyses the performance of the system in an environment where the multipath signals on each of the diversity branches of a RAKE receiver have varying fading characteristics. This scenario would typically describe urban environments where a large number of multipath echoes are present, each with different fading statistics resulting from the non-homogeneous propagation paths seen by each multipath echo.

I. INTRODUCTION

Mobile and wireless communication networks are playing an ever increasing role in the supply of a number of heterogeneous services to both mobile and fixed users. Considering the service offerings envisaged by UMTS and IMT-2000, it is clear that a single network will supply the services required by both mobile and fixed subscribers.

The first assumption used in determining the performance of cellular networks is that all users require the same data rate. This leads to the assumption that all users utilize spreading sequences of the same length. Clearly this is no longer the case as the W-CDMA proposal by ETSI for UMTS defines a procedure to use spreading sequences with different lengths for different service rates.

Secondly, the performance analysis of cellular systems in general do not differentiate between users on the grounds of their spatial location and mobility. With the convergence of mobile and fixed networks this assumption is becoming increasingly invalid. In fact, even in current mobile networks a single base station may service areas in which there are users with very high levels of mobility as well as users that can be described as fixed for the duration of a call.

The concept of spatial filtering or beam steering techniques and its application to cellular CDMA systems have also been receiving increased attention [1]. Specifically antenna arrays have been shown to increase the capacity of cellular systems and have been earmarked for inclusion in 3G communication systems. The increase in BER performance that can be achieved using antenna arrays is calculated in [2], [3] for a Rayleigh fading channel.

In this paper, the existing literature outlined above is extended by considering a cellular system model where the multipath signals received at the base station are character-

ized by independent Nakagami fading with different fading statistics. Specifically the results of [4] are extended by firstly establishing a connection between the location of the user and the propagation path and secondly through the inclusion of an antenna array in the system analysis. The link between the fading parameters and the location of the user is based on the work presented in [5]. Incorporating this link between the physical surroundings of a mobile user and the channel model assumed for the user is important as measured data have shown that multipath fading statistics in urban areas are non-identical. Whereas the effects of the non-identical propagation paths may be averaged when large beamwidth base station antennas (such as an omni-directional antenna) are used, the introduction of narrow-beam adaptive antenna arrays for spatial filtering into DS-CDMA systems would conceptually amplify any difference in performance, and therefore capacity, that would result from different multipath fading statistics for different propagation paths.

The remainder of the paper is organized as follows. In section II, a channel model is established. The BER performance of a DS-CDMA system is calculated analytically in section III. Numerical performance results for various physical scenarios that may be found in cellular systems are presented in section IV, and finally some concluding remarks are given in section V.

II. MODELING THE PROPAGATION CHANNEL

The principle of modeling the propagation channel as a function of angle has been proposed by numerous authors. In this paper a simpler approach that yields a first order approximation of an "average" channel as seen from the base station for macro and micro cellular systems is used as presented in [5]. In this model, the local scattering areas around mobile users are assumed to be the dominant cause of scattering and this scattering area is modeled using a Gaussian bell shaped distribution of scatterers. Furthermore, it is shown that the angular distribution of users significantly influence the Direction of Arrival (DOA) of signals at the base station in the case of the cellular uplink. Therefore, a probability density function (pdf) for the angle of arrival of signals at the base station must take into account both factors, and given by

$$p_{\theta_b}(\theta_b) = \frac{A}{2\sqrt{2\pi}\sigma} e^{\frac{D^2(\cos^2\theta_b - 1)}{2\sigma^2}} \operatorname{erfc}\left(\frac{-D\cos\theta_b}{\sqrt{2}\sigma}\right) \quad (1)$$

with A denoting a normalizing constant such that $\int_0^{2\pi} p_{\Theta_b}(\theta_b) d\theta_b = 1$, $\text{erfc}(x)$ denoting the complementary error function, D denoting the base station mobile user separation and σ^2 a site specific propagation factor with values $\sigma = 0.34R$ for non line-of-sight micro cellular conditions; $\sigma = 0.2R$ for line-of-sight micro cellular conditions and $\sigma = 0.1R$ for macro cellular conditions, where R is equal to the cell radius. Several proposals describing the angular distribution of users, $p_{\Theta_b}(\theta_b)$, have also been proposed [5]. In this paper, a uniform angular distribution for mobile users is assumed, even though the results presented are easily extendable to non-uniform distributions using the results in [6].

In addition to the model of the DOA, the fading effects of each multipath echo arriving with a certain DOA needs to be taken into account. Typically, all the received signals are modeled as having either a Rayleigh, Rician or a constant m , Nakagami distributions. In this paper, the Nakagami distribution will be used to describe the fading envelope as it is well known that the Nakagami distribution is equivalent to the Rayleigh distribution when $m = 1$, the One-Sided Gaussian distribution when $m = 0.5$ and because it can also model Rician distributions with sufficient accuracy by setting

$$m = \frac{1}{1 - \left(\frac{\mathcal{K}}{1+\mathcal{K}}\right)^2} \quad (2)$$

where \mathcal{K} denotes Rice factor (average direct power/average scattered power). Also in [7] it is shown that the Nakagami model can be used to accurately describe the fading behavior of multipath signals. Specifically it is shown that the Nakagami distribution can be used to describe the varying physical scattering processes.

Furthermore, it is assumed that the propagation path may change as a function of the DOA of signals. Also, in the experimental study in [8], the urban propagation channel is modeled as a Rician channel with varying \mathcal{K} parameter. In all relevant cases considered in this study (typical urban and bad urban), the cumulative distribution function (cdf) of the \mathcal{K} parameter measured over the three strongest paths received is close to the cdf of a Gaussian pdf or two-sided exponential distribution. Based on these results, two models that may be used to create a relationship between m and θ are proposed.

A. Exponential Fading Distribution

Firstly, a relationship between m and θ based on a generalized exponential function by setting m in (2) equal to

$$m(\theta) = m_0 e^{-\delta_m(|\theta - \theta_0|)} \quad m(\theta) > 0.5 \quad (3)$$

where m_0 denotes the Nakagami parameter of the main received path, θ_0 the DOA of the main received signal path and δ_m a parameter to control to the increase in the severity of the fading as a function of the angular spread of the multipath signals arriving at the base station. When $\delta_m = 0$, this model represents a standard, constant m Nakagami fading channel. The changes in m as a function of θ

(that is the value of δ_m) will depend on the local scattering environment.

B. Gaussian Fading Distribution

As is shown in [5], the local scattering elements surrounding a mobile user can be described by a Gaussian distribution. Based on this distribution and the results in [8], the relationship between m and θ can be written as

$$m(\theta) = m_0 e^{-\frac{(\theta - \theta_0)^2}{2\sigma_m^2}} \quad m(\theta) > 0.5 \quad (4)$$

In (4), the fading parameters of the multipath components as a function of θ is controlled by the parameter σ_m , the variance of the fading parameter. Setting σ_m equal to ∞ will yield a constant m Nakagami fading channel. Decreasing σ_m will yield non-constant fading distributions with the fading effects becoming increasingly severe as the DOA of the multipath signal is removed from the DOA of the main received signal path.

In the case of both the exponential and Gaussian models, the DOA of the multipath signals θ , is determined by (1).

III. PERFORMANCE ANALYSIS

We assume the uplink of a single cell DS-CDMA system with K active users. The output of the transmitter of user k can be written as

$$s^{(k)}(t) = \sqrt{2P} a^{(k)}(t) b^{(k)}(t) \cos(\omega_c t + \phi^{(k)}) \quad (5)$$

where P denotes the average transmitted signal power, $b^{(k)}(t)$ denotes binary data with bit period T seconds, $a^{(k)}(t)$ denotes a random binary spreading sequence with chip period T_c seconds and length $N = T/T_c$. Also, standard Binary Phase Shift Keying (BPSK) modulation is used with carrier frequency ω_c rad/s and unknown carrier phase $\phi^{(k)}$, a random variable uniformly distributed over $[0, 2\pi)$. The transmitted signal propagates over a radio channel modeled as a Nakagami fading, time invariant, discrete multipath channel with equivalent low-pass response

$$h^{(k)}(\tau) = \sum_{l=0}^{L_p-1} \beta_l^{(k)} e^{j\psi_l^{(k)}} \delta[\tau - \tau_l^{(k)}] \quad (6)$$

Each of the L_p received paths are characterized by the variable $\beta_l^{(k)}$, a Nakagami distributed random variable denoting the strength of path l from user k , $\psi_l^{(k)}$ uniformly distributed over $[0, 2\pi)$ and denoting the phase shift of path l from user k and $\tau_l^{(k)}$, uniformly distributed over $[0, T)$ and denoting the propagation delay of path l from user k . Now, assuming coherent operation the received signal can be written as

$$r(t) = \sum_{k=1}^K \sum_{l=0}^{L_p-1} \sqrt{2P} \beta_l^{(k)} a^{(k)}(t) b^{(k)}(t) \cos(\omega_c t + \phi^{(k)} + \psi_l^{(k)}) + \eta(t) \quad (7)$$

with the decision variable on the n^{th} diversity branch of the RAKE receiver equal to

$$\zeta = \sum_{n=1}^{L_R} \left\{ S^{(n)} + I_{mai}^{(n)} + I_{si}^{(n)} + I_{ni}^{(n)} \right\} \quad (8)$$

where $S^{(n)}$ denotes the desired received signal

$$S^{(n)} = \sqrt{\frac{P}{2}} T b_0^{(1)} \beta_n^{(1)2} \|\bar{w}_1\| \quad (9)$$

with $\beta_n^{(1)}$ the weight of the n^{th} branch of the RAKE receiver and where $I_{mai}^{(n)}$ denotes the multiple access interference present in the cell,

$$I_{mai}^{(n)} = \sqrt{\frac{P}{2}} \sum_{k=2}^K \sum_{l=0}^{L_p-1} \beta_n^{(1)} \beta_l^{(k)} \|\bar{w}_k\| \cdot \bar{\mathcal{R}}_{k1} \{ b_{-1}^{(k)} R_{k1}[\tau_{nl}^{(k)}] + b_0^{(k)} \hat{R}_{k1}[\tau_{nl}^{(k)}] \} \cos(\psi_{nl}^{(k)}) \quad (10)$$

$I_{si}^{(n)}$ denotes the self interference present in the cell,

$$I_{si}^{(n)} = \sqrt{\frac{P}{2}} \sum_{l=0, l \neq n}^{L_p-1} \beta_n^{(1)} \beta_l^{(1)} \|\bar{w}_1\| \cdot \bar{\mathcal{R}}_{11} \{ b_{-1}^{(1)} R_{11}[\tau_{nl}^{(1)}] + b_0^{(1)} \hat{R}_{11}[\tau_{nl}^{(1)}] \} \cos(\psi_{nl}^{(1)}) \quad (11)$$

$I_{ni}^{(n)}$ denotes the Additive White Gaussian Noise (AWGN) interference

$$I_{ni}^{(n)} = \int_{nT_c}^{T+nT_c} \eta(t) \beta_n^{(1)} a^{(1)}(t-nT_c) \cos(\omega_c t + \psi_n^{(1)}) dt \quad (12)$$

with $b_0^{(1)}$ being the information bit to be detected, $b_{-1}^{(1)}$ the preceding bit, $\tau_{nl}^{(k)} = \tau_l^{(k)} - \tau_n^{(1)}$, $\psi_{nl}^{(k)} = \psi_l^{(k)} - \psi_n^{(1)}$ and

$$\begin{aligned} R_{k1}(\tau) &= \int_0^T a^{(k)}(t-\tau) a^{(1)}(t) dt \\ \hat{R}_{k1}(\tau) &= \int_\tau^T a^{(k)}(t-\tau) a^{(1)}(t) dt \end{aligned} \quad (13)$$

and

$$\bar{\mathcal{R}}_{k1} = \frac{\text{Re}[\bar{w}_1^H \bar{w}_k]}{\|\bar{w}_1\| \|\bar{w}_k\|} \quad (14)$$

with $()^H$ denoting the Hermitian transpose and \bar{w}_k the array manifold vector or steering vector optimizing the response of the antenna array for user k . The antenna array elements are assumed to be sufficiently closely spaced to ensure that the signals received at each antenna element is highly correlated. More specifically, it is assumed that the correlation between the signals received at each element of the antenna array is greater than 0.8. If this correlation factor is lower, the antenna pattern synthesized by the adaptive antenna array will exhibit grating lobes. This would enable digital beamforming techniques to be used to implement a spatial filter.

As is used generally [9], it is assumed that all the interference terms present in (8) are Gaussian distributed. This assumption has been shown to be accurate, even for small values of K when the BER is 10^{-3} or greater. Therefore, expanding on the results in [4] to include the effects of antenna arrays as shown above, the variances of each interference term can be written as

$$\sigma_{mai,n}^2 = \frac{E_b T}{6N} \{\beta_n^1\}^2 \sum_{k=2}^K \|\bar{w}_k\|^2 E\{\bar{\mathcal{R}}_{k1}^2\} \sum_{l=0}^{L_p-1} \Omega_l^k \quad (15)$$

$$\sigma_{si,n}^2 = \frac{E_b T}{4N} \{\beta_n^1\}^2 \|\bar{w}_1\|^2 E\{\bar{\mathcal{R}}_{11}^2\} \sum_{l=0}^{L_p-1} \Omega_l^1 \quad (16)$$

$$\sigma_{ni,n}^2 = \frac{T\eta}{4} \{\beta_n^1\}^2 \quad (17)$$

yielding a total interference term of

$$\sigma_T^2 = \sum_{n=0}^{L_R-1} (\sigma_{mai,n}^2 + \sigma_{si,n}^2 + \sigma_{ni,n}^2) \quad (18)$$

where L_R denotes the number of branches in the RAKE receiver. Furthermore, the output of the RAKE combining receiver can be written as

$$U_S = \sqrt{\frac{E_b T}{2}} \|\bar{w}_1\| \sum_{n=0}^{L_R-1} \{\beta_n^{(1)}\}^2 \quad (19)$$

The variance of the Nakagami fading parameters of each user, $E\{\{\beta_l^{(k)}\}^2\}$ is equal to the average signal power received from that user, $\Omega_l^{(k)}$. We assume that the multipath signals are characterized by an exponential Multipath Intensity Profile (MIP), i.e.

$$\Omega_l^{(k)} = \Omega_0^{(k)} e^{-l\delta}, \quad \delta > 0 \quad (20)$$

where $\Omega_0^{(k)}$ is the average signal strength corresponding to the first incoming path of user k and δ is the rate of average power decay. Then, the variance of the total interference is

$$\begin{aligned} \sigma_T^2 &= E_b T \Omega_1^{(1)} \left[\frac{\sum_{k=2}^K \|\bar{w}_k\|^2 E\{\bar{\mathcal{R}}_{k1}^2\} \Omega_1^{(k)} \sum_{l=0}^{L_p-1} e^{-l\delta}}{6N \Omega_1^{(1)}} \right. \\ &\quad \left. + \frac{\|\bar{w}_1\|^2 E\{\bar{\mathcal{R}}_{11}^2\} \sum_{l=0}^{L_p-1} e^{-l\delta}}{4N} + \frac{\eta}{4E_b \Omega_1^{(1)}} \right] \cdot \sum_{n=0}^{L_R-1} \{\beta_n^{(1)}\}^2 \end{aligned}$$

Furthermore, defining

$$S = \frac{1}{\Omega_0^{(1)}} \sum_{n=0}^{L_R-1} \{\beta_n^{(1)}\}^2 \quad (21)$$

the output Signal to Noise Ratio (SNR), $U_s^2/2\sigma_T^2$, can be written in compact form as $\sigma_0 S$ where

$$\begin{aligned} \sigma_0 &= \left[\frac{2 \sum_{k=2}^K E\{\bar{\mathcal{R}}_{k1}^2\} \Omega_0^{(k)} \sum_{l=0}^{L_p-1} e^{-l\delta}}{3N \Omega_0^{(1)}} \right. \\ &\quad \left. + \frac{E\{\bar{\mathcal{R}}_{11}^2\} \sum_{l=0}^{L_p-1} e^{-l\delta}}{N} \right. \\ &\quad \left. + \frac{\eta}{\|\bar{w}_1\|^2 E_b \Omega_0^{(1)}} \right]^{-1} \end{aligned} \quad (22)$$

assuming that $\|\bar{w}_k\|^2$ is equal for all k .

For coherent demodulation, the BER can be expressed as

$$\bar{P}_e = \frac{1}{2\pi} \int_0^\infty \mathcal{D} (C \sin(B - A) + \mathcal{D} \sin(A)) dt \quad (23)$$

where

$$A = \sum_{l=0}^{L_R-1} m_l \tan^{-1} \left(\frac{t}{\lambda_l} \right) \quad (24)$$

$$B = \frac{1}{2} \tan^{-1} \left(\frac{t}{\sigma_0} \right) \quad (25)$$

$$C = \frac{\sqrt{\sigma_0}}{(t^2 + \sigma_0^2)^{1/4}} \quad (26)$$

and

$$\mathcal{D} = \left(\prod_{l=0}^{L_R-1} \left[1 + \left(\frac{t}{\lambda_l} \right)^2 \right]^{m_l/2} \right)^{-1} \quad (27)$$

and now, $\lambda_l = \Omega_0^{(1)} m_l / \Omega_l^{(1)}$. This integral can be computed numerically. Furthermore, m_l is assumed to be a function of the DOA of the signal at the base station and is described by (3) and (4).

IV. NUMERICAL RESULTS

In order to determine the performance of the cellular CDMA system Monte-Carlo simulation techniques are used. Therefore, L multipath signals are generated for each user with DOA's distributed according to (1) during each trial of the Monte-Carlo process. Following this, the fading parameter m is determined according to either the Exponential or Gaussian models defined in (3) and (4) where θ is the DOA of each multipath signal. As examples, the decay rate of the Exponential model is set to $\delta_m = 1$ and the standard deviation of the Gaussian model is set to $\sigma_m = 0.6$. These choices of δ_m and σ_m yields fading models where, in the N-LOS case, the fading on the various RAKE receiver branches vary from Rayleigh to One-Sided Exponential, the two distributions used most frequently to describe the fading of signals in bad urban N-LOS propagation environments. In the LOS case, the main received signal path is equivalent to a Rician fading signal with \mathcal{K} parameter of 10 dB, with the most severely faded signal equivalent to a Rician faded signal with $\mathcal{K} = 5.3$ dB in the case of the exponential model. The main received signal fading parameters is equivalent to the fading experienced by at least 50% of signals received in typical urban environments, with the most severe fading being equivalent to that experienced by 10% of the signals in a typical urban environment. For the Gaussian model, the most severe fading is approximately Rayleigh, also a common fading assumption. Furthermore, it is assumed that the $L = 5$ resolvable paths are received from each user in the system. These values yield fading parameters for the system depicted in Figure 1. From the figure it is clear that, as expected, LOS environments exhibit less fading than N-LOS environments. Furthermore, it can also be seen that the Gaussian model results in more

severe fading condition than the Exponential model. This is clearly a function of the choice of δ_m and σ_m , but in general, the exponential distribution decays slower than the Gaussian distribution.

In order to determine the BER performance of the system, it is assumed that $K = 10$ users are active in the cell, each with random spreading sequences of length $N = 63$ and channel model as outlined above. Comparing the BER performance of the system for the two varying fading models outlined above and the standard assumption of equal m -values on all branches of the RAKE receiver, it is found that the Gaussian model yields worse performance for LOS and N-LOS scenarios (see Figure 2) than the other two models. Also, the effect of varying m -values are greater when the number of taps in the RAKE receiver is increased.

In Figure 2, a constant multipath model is assumed ($\delta = 0$). This assumption may be used to approximate bad urban scenarios [8] whereas typical urban environments may be better approximated by exponential multipath decay models [8]. Figure 3 shows the performance of the system for these two cases. From the figure it is clear that varying m -values have a greater influence on the change in BER performance when a constant multipath model is assumed. This is intuitively correct as the multipath components received with more severe fading has less power in the case of the exponential model.

Further analyzing the constant multipath scenario as found in bad urban scenarios for the case when larger antenna arrays are used at the base station further underlines the need to consider varying fading parameters for the branches of the RAKE receiver. Figure 4 show the BER performance for the system described above with $M = 1, 3, 5$ element arrays respectively. As can be expected, the performance of the system increases as M increases. Also, as in the previous cases, the Gaussian model yields the worst system performance. Furthermore, the system performance is better when the system operates under LOS conditions as can be seen when Figures 4 and 5 are compared. In addition to the better system performance, the effect of the varying m values is more pronounced in the LOS case. It is also clear that the effect of varying m -values are greater when the number of elements in the beam forming array is increased.

V. CONCLUSIONS

The results presented in this paper clearly indicate that the non-uniform propagation conditions encountered in cellular networks can severely influence system performance and capacity. Specifically, the non-uniform propagation conditions degrade system performance in the case of bad urban conditions where a large number of equal strength multipath echos are received with a RAKE receiver. Finally, the numerical results show clearly that the constant m fading assumption will yield a lower bound on the BER performance, outage performance and system capacity of a DS-CDMA system. Therefore, especially in the case of non-interference limited systems (small K , large E_b/N_0), varying m -values must be considered in order to obtain an

accurate measure of the true system capacity.

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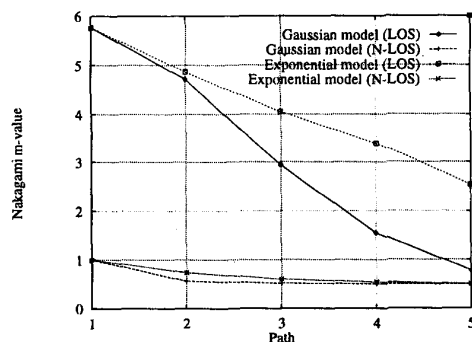


Fig. 1. Fading parameters of multipath echos received at the base station ($\delta_m = 1, \sigma_m = 0.6$).

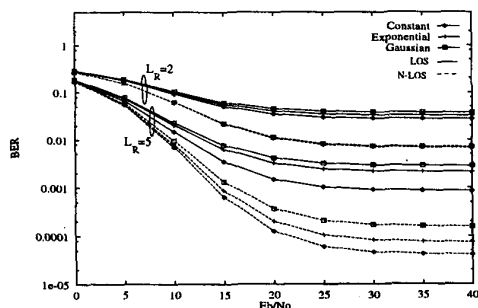


Fig. 2. BER performance as a function of E_b/N_0 for a cellular system with a $L_R = 2$ and $L_R = 5$ tap RAKE receiver ($L = 5, \delta = 0, M = 1$).

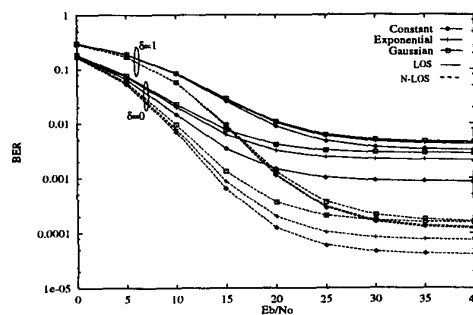


Fig. 3. BER performance as a function of E_b/N_0 for a cellular system with a $\delta = 0$ and $\delta = 1$ ($L = 5, L_R = 5, M = 1$).

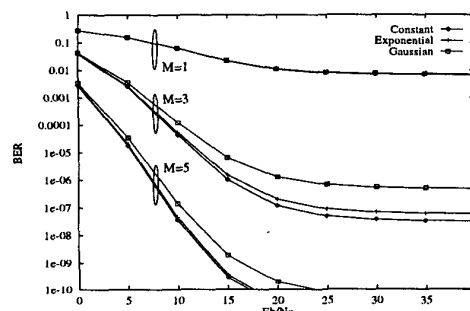


Fig. 4. BER performance as a function of E_b/N_0 for a cellular system with a $M = 1, M = 3$ and $M = 5$ ($\delta = 0, L = 5, L_R = 2$, LOS propagation).

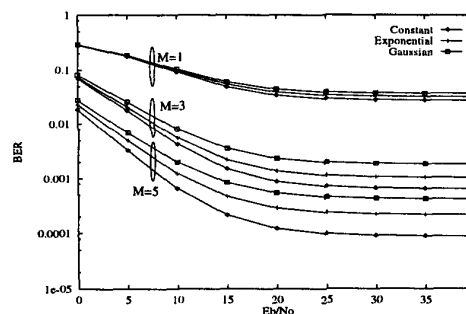


Fig. 5. BER performance as a function of E_b/N_0 for a cellular system with a $M = 1, M = 3$ and $M = 5$ ($\delta = 0, L = 5, L_R = 2$, N-LOS propagation).