Spatial Considerations in the Design and Simulation of Cellular Multiple Access Systems

Michiel P. Lötter † and Pieter van Rooyen *

† Alcatel Altech Telecoms, P.O. Box 286, Boksburg, South Africa 1460

* Department of Electrical and Electronic Engineering, University of Pretoria, Pretoria, South Africa

Abstract—**In this paper, some spatial considerations in the design and simulation of cellular communication systems are discussed. In particular, new probability density functions (pdf) are used to describe the location of users in a cellular system and analytical expressions for the pdf of the angle of arrival (AOA) of signals at the subscriber are derived for systems employing directional antennas or smart antennas. These expressions can be used to derive analytic expressions for the bit-error-rate performance of Space Division Multiple Access (SDMA) communication systems.**

I. INTRODUCTION

The rapid growth in wireless communication services and networks has placed enormous strain on available resources, i.e.. bandwidth. The mere fact that the Federal Communications Commission (FCC) in the United States was able to auction 90 MHz of spectrum allocated to PCS for $17.2 billion is clear evidence of this [1]. Added to this is the additional strain placed on the available spectrum by services such as Wireless Local Loop (WLL), Cordless Phones and broadband access services [2], [3], [4]. This path of rapid evolution has brought with it some of the biggest engineering challenges to date. A few of years ago, the design of a communication system capable of operating in multiple propagation environments, delivering a broad range of services each with its own quality requirements on a global scale to a person with a single handheld terminal [4] would have sounded far fetched. Today this is reality. The definition of a Universal Mobile Telephone System (UMTS) is currently receiving a great deal of attention from standardization bodies such as the International Telecommunications Union, and current evolution paths predict the availability of UMTS services early in the next century [5], [4]. This process has been the driving force behind the design of new coding [1], [6], [7], modulation [8], [9], [10], planning [11], [12] and access methodologies [13], [14], [15], [16]. Specifically, more dimensions are added to the cellular engineering problem. Starting with Frequency Division Multiple Access (FDMA) systems, cellular networks have seen the introduction of Time Division Multiple Access (TDMA) systems, Code Division Multiple Access (CDMA) systems [17] and today, SDMA systems [18], [19], [20].

SDMA systems rely on the use of adaptive narrowbeam antennas and the non-homogeneous distribution of users in a cellular system to increase system capacity. In particular, adaptive antenna arrays have been receiving a lot of attention in literature [21], [22], [23]. The basic principle of a SDMA system, is that users in the same cell can use the same time, frequency and code resources if they are physically separated, by servicing each user with a spot-beam antenna. As these systems rely on the spatial information of users in a cell to increase system performance, assumptions made in this regard when analysing SDMA communication systems are extremely important. In this paper, the basic spatial assumptions required to analyse a SDMA system are discussed, and various analytical expressions describing these assumptions are derived.

In the next section, assumptions relating to the location of subscribers, as well as the location of scattering elements around the subscriber are discussed. These form the basis of the analysis of a SDMA system as, for instance, an assumption that subscribers are uniformly distributed throughout a cell would yield a greater increase in BER performance than a system where all subscribers are clustered into a small area. In section III, the pdf of the angle of arrival of signals at a mobile user in the cell is calculated, and finally in section IV it is shown where these expressions can be used in the analysis of SDMA based cellular systems.

II. SYSTEM DESCRIPTION

A. Physical Parameters

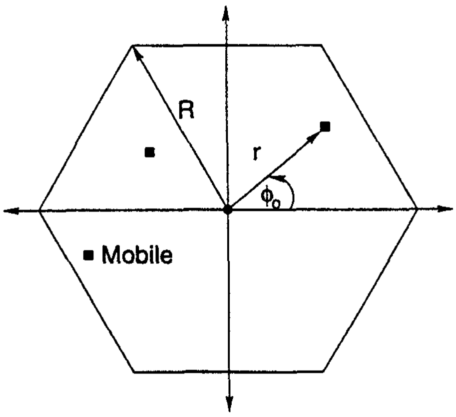


Fig. 1. Definition of spatial variables.

The cellular multiple access system is assumed to be constituted by a number of non-overlapping hexagonal cells, each of radius R. Each cell has a base station located at its centre, with mobile users distributed throughout the cell. The position of a mobile user in the cellular structure (reference and

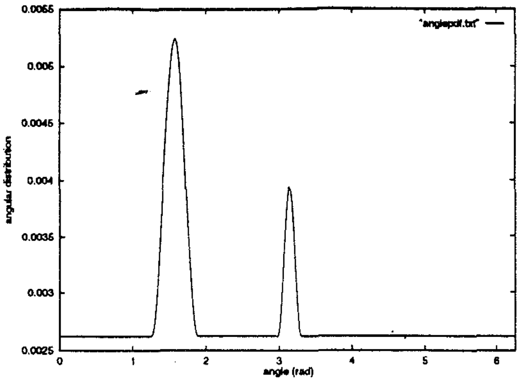
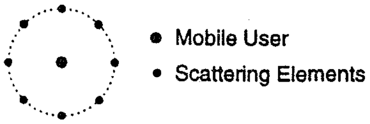


Fig. 2. Example pdf of the angular distribution of subscribers.

(a) Ring of Scattering Elements



(b) Disc of Scattering Elements

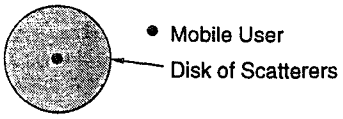


Fig. 3. Modeling of scattering elements as a ring of scatterers (a) or a contiguous disc of scatterers.

adjacent cells) is fully defined by its distance from the reference base station r, and its angle, ϕ_o, measured from some reference, both of which can be considered random variables. This is shown in Figure 1. As the performance of a SDMA system depends on the spatial distribution of subscribers, true pdf's for both the angular and distance distributions of subscribers are required in order to calculate the performance of a SDMA system. These are, however, not available in the literature and various approximations such as uniform angular and distance distributions [24] and modified gaussian distributions [25] are used. These assumptions are either not a good representation of the real world scenario or are somewhat inflexible as it is clear that these distributions will vary from situation to situation. In this paper, the same general pdf shall be used to describe both the angular and distance distributions of subscribers. In terms of the angular distribution of subscribers, the pdf can be written as

$$\begin{aligned} p_{\Phi_o}(\phi_o) &= \frac{1}{A_{norm}}\left[1+\sum_{l=1}^{N_{peak}}\gamma_l\left[\mathrm{rect}\left(\frac{w_l\phi_o}{\pi}-\alpha_l\right)\right.\right. \\ &+ \left.\mathrm{rect}\left(\frac{w_l\phi_o}{\pi}-\alpha_l-2\pi\right)\right] \\ & cos^2(w_l\phi_o-\alpha_l)\Big] \end{aligned} \tag{1}$$

where A_{norm} is a normalizing factor to ensure that $\int_0^{2\pi} p_{\Phi_o}(\phi_o)d\phi_o = 1$ and N_{peak} is the number of peaks in the pdf. This factor is a measure of the clustering of subscribers in a cell. Clearly, if $N_{peak} = 0$, eq. 1 denotes a uniform angular subscriber distribution. Furthermore, rect(x) is defined as

$$\mathrm{rect}(x) = \begin{cases} 1 & |x| < \frac{1}{2} \\ 0 & |x| > \frac{1}{2} \end{cases} \tag{2}$$

with w_l an integer controlling the width of peak l. Typically, values for w_l will be chosen to yield angular peaks of different maximum and minimum widths. For instance, if $w_l = 10$ the width of peak l in the pdf of the angular distribution of subscribers will be $\pi/10$ rad. The angular location of peak l is given by α_l, whilst the relative size of each peak is determined by γ_l (typically values between 0 and 1). Figure 2 depicts an example of an angular distribution pdf with $N_{peak} = 2, w_1 = 5, w_2 = 10, \alpha_1 = pi/2, \alpha_2 = pi, \gamma_1 = 1$ and $\gamma_2 = 0.5$. By using the variable transformation

$$r = \phi_o\frac{3R}{2\pi} \tag{3}$$

equation 1 can also describe the distance distribution of subscribers. Using the above equations, different scenarios, such as clustering of subscribers, can be represented.

In addition to the above, each subscriber is assumed to be surrounded by a large number of scattering elements. Several approaches to the description of these scattering elements exist. Firstly, the scattering elements can be described as a ring of scatterers around the mobile user (Fig 3a) [21] or secondly, the scattering elements can be considered to be a disc of scatterers around the mobile user (Fig 3b) [26]. However, in [27] it is shown that scattering elements can be best described by a gaussian bell shape as is shown in Figure 4. In this case it is assumed that most scatterers are situated close to the mobile user, with the density of scatterers decreasing as the distance from the mobile is increased.

In Figure 4 the mobile M is separated from the bases station B by an arbitrary distance, D. The base station employs a directional antenna with beamwidth 2α to illuminate the mobile, as well as a portion of the scatterers around the mobile. In this case, the density of the scatterering elements at a distance r_s and angle θ_s from the mobile, can be described by the bivariate gaussian distribution

$$\begin{aligned} P_{scat}(r_s,\theta_s) &= \frac{1}{2\pi\sigma^2}e^{\frac{-r_s^2(\cos^2\theta_s+\sin^2\theta_s)}{2\sigma^2}} \\ &= \frac{1}{2\pi\sigma^2}e^{\frac{-r_s^2}{2\sigma^2}} \end{aligned} \tag{4}$$

where θ_s is measured from the horizontal in Figure 4.

B. Antenna Parameters

Each base station employs a MxN element planar antenna array with parameters as shown in figure 5. Elements are considered to be isotropic radiators with complex gain factors I_{mn}, where m and n denote the row and column number of the specific element respectively. Note that by setting $I_{mn} = 0$ for certain values of m and n, the planar array can represent various array geometries. For instance, setting $I_{mn} = 0 \; \forall \; m > 1$ yields a standard linear array.

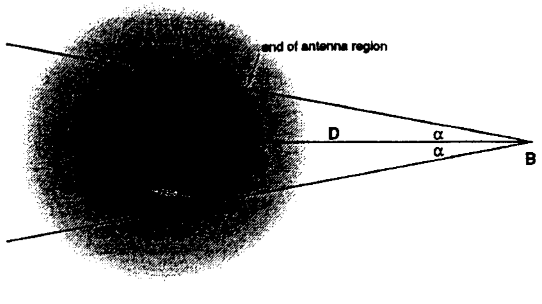


Fig. 4. Modeling of scattering elements using a Gaussian approach.

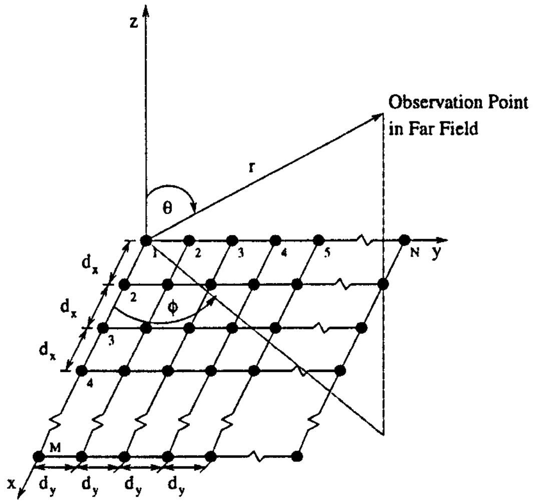


Fig. 5. Generic structure of planar array at each base station.

The radiation pattern of an antenna array is described by the Array Factor (AF), and the radiation pattern of a single radiating element. As we assume isotropic radiating elements, the radiation pattern is directly proportional to the AF. In the case of the array depicted in Figure 5, the AF can be seen as the product of the AF's of N linear arrays consisting of M elements each, that is [23], [28]

$$AF = \sum_{n=0}^{N-1} e^{jknd_y \sin\theta sin\phi + n\beta} \sum_{m=0}^{M-1} I_{mn} e^{jkmd_x \sin\theta\cos\phi} \tag{5}$$

where $k = 2\pi/\lambda$, $n\beta$ denotes the progressive phase shift between each of the M element linear arrays and the first M element array and I_{mn} denotes the amplitude of and progressive phase relative to the first element in the M-element array. A typical radiation pattern is depicted in Figure 6, for the case $M = 5, N = 1, I_{mn} = 1, \beta = 0$. From the figure, it is clear that the main lobe of the radiation pattern is directed towards $\phi = 90°$ and $\phi = 270°$, and that the width of the main lobe (null-to-null) is approximately 28°.

III. PROBABILITY DENSITY FUNCTION OF THE ANGLE-OF-ARRIVAL.

Once assumptions on the location of subscribers have been made (see eq. 1), and the beamwidth of the antenna structure is known (see eq. 5), the pdf of the AOA of signals at the mobile can be determined. The general assumption used is that a mobile user may receive reflections from any direction with equal probability. This assumption is only true if the antenna beam illuminates all scattering elements around the mobile [26]. Clearly this is not the case for narrow beam antennae as is shown in Figure 4. In order to calculate the pdf of the AOA's, the cumulative distribution of scattering elements as a function of θ_s is required. This cumulative distribution, or the addition of more scattering elements as a function of angle is clearly

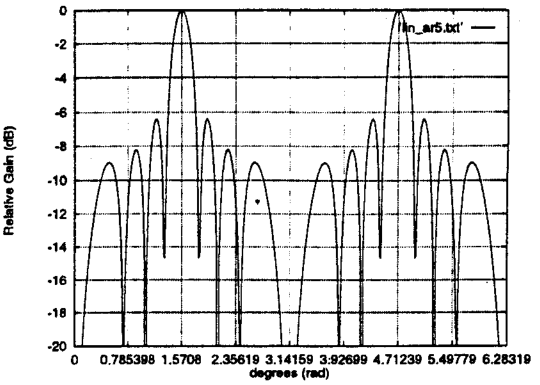


Fig. 6. Radiation pattern of a linear array with $M = 5, N = 1, I_{mn} = 1, \beta = 0$.

$$W_{\theta_s} = \int_0^{\theta_s} \int_0^{r_{end}} \frac{1}{2\pi\sigma^2} e^{-\frac{r_s^2}{2\sigma^2}} dr_s d\theta \tag{6}$$

where r_{end} denotes the distance to where the antenna beam illuminates scatterers (see Figure 4). The *end of antenna region* can be divided into the following three distinct areas

$$r_{end} = \begin{cases} \frac{D \tan\alpha \tan\theta}{\sin\theta(\tan\alpha + \tan\theta)} & 0 \le \theta \le \pi/2 \\ \frac{D\tan\alpha}{\cos\theta(\tan\alpha+\tan\theta)} & \pi/2 < \theta < \pi - \alpha \\ \infty & \pi - \alpha \le \theta \le \pi \end{cases} \tag{7}$$

The probability of receiving a signal with a certain AOA is then directly related to the density of scatterers in the specific direction, or the derivative of eq. 6 with respect to θ_s. Therefore

$$p_{\Theta_s}(\theta_s) = \frac{d}{d\theta_s} W_{\theta_s} = \int_0^{r_{end}} \frac{A}{2\pi\sigma^2} e^{-\frac{r_s^2}{2\sigma^2}} dr_s \tag{8}$$

where A is a normalizing constant such that $\int_0^{2\pi} p_{\Theta_s}(\theta_s) d\theta_s = 1$. Equation 8 can be simplified by algebraic manipulation to

$$p_{\Theta_s}(\theta_s) = \frac{A}{2\sqrt{2\pi}\sigma} \text{erf}\left(\frac{r_{end}}{\sqrt{2}\sigma}\right) \tag{9}$$

where

$$\text{erf(x)} = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt \tag{10}$$

is the well known error function. The normalizing constant can be calculated by noting that, by definition, $\int_0^{2\pi} \int_0^{\infty} W_{\theta_s} dr_s d\theta_s = 1$ and that A would be the ratio of the illuminated to the non-illuminated scattering elements. Furthermore, using the symmetry of the Figure 4, the pdf of the AOA can be shown to be the same for $0 \le \theta_s \le \pi$ and $-\pi \le \theta_s \le 0$.

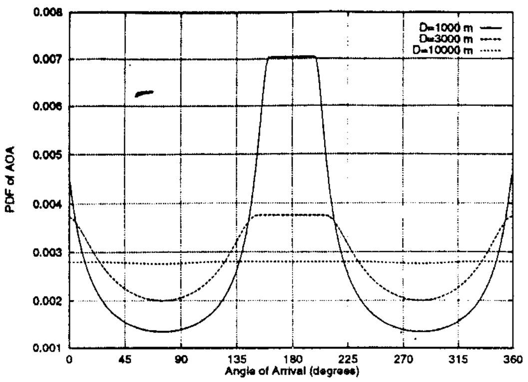


Fig. 7. PDF of the AOA with D=1000m, D=3000m and D=10000m.

The pdf of the AOA for the case where the base station and the mobile are separated by D=1,3 and 10 km, with a beamwidth of 28°, and a scatterer variance, σ of 1000 is shown in Figure 7. From the figure it is clear that the pdf of the AOA is a function of the separation between the base station and the mobile. When the mobile is far from the base station, nearly all scattering elements are illuminated by the antenna and the pdf approximates a uniform distribution. On the other hand, when the mobile is close to the base station only a fraction of the scattering elements are illuminated and the pdf is no longer uniform.

It should also be clear from the above discussion that the pdf described by equation 9 can be offset by an angle ϕ_o to describe the pdf of the AOA's for different subscribers in the same axii. This offset value is a random variable described by the pdf of equation 1.

IV. CONCLUSIONS

In this paper it was shown how the pdf of the AOA of signals at a mobile user illuminated by a narrow beam antenna can be calculated under some physical assumptions about the location of subscribers and for general antenna geometries simulated by a planar array. The analytical expressions derived have the advantage that they are general and flexible and can be used to represent various subscriber distributions, such as clustering. The pdf of the AOA is required to calculate various parameters of a cellular access system, such as BER, outage probability and Doppler Spectrum. It is also important to note that the pdf of the AOA can not be assumed to be equal for all subscribers under all circumstances, as it is a function of the separation between the base station and the mobile.

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