Space Division Multiple Access for Cellular CDMA

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Abstract— **Space Division Multiple Access (SDMA) will form an important part of the new Wideband Code Division Multiple Access (WCDMA) standard that will realise the Universal Mobile Telephone System (UMTS). This paper addresses a few issues of importance when SDMA techniques are used in a cellular CDMA system. Firstly a theoretical analysis of a SDMA/CDMA system is performed. The analysis is focused on a single cell, multipath Rayleigh fading scenario. As system performance measure Bit Error Rate (BER) is used as criteria to investigate the influence of user location and number of antennas. An important parameter in a SDMA system is the antenna array element spacing. In our analysis a Uniform Linear Array (ULA) is considered and a measure is defined to determine the optimal antenna element spacing in a CDMA cellular environment. Normally the mobile users in a cell are assumed to be uniformly distributed in cellular performance calculations. To reflect a more realistic situation, we propose a novel probability density function for the non-uniform distribution of the mobile users in the cell. It is shown that multipath, even with antenna arrays, reduces the system performance substantially.**

I. INTRODUCTION

The rapid growth in wireless communication services and networks have placed enormous strain on available natural resources, ie. bandwidth. The mere fact that the Federal Communications Commission (FCC) in the United States was able to auction 90 MHz of spectrum allocated to PCS for $17.2 billion is clear evidence of this [1]. Added to this are the additional strain placed on the available spectrum by services such as Wireless Local Loop (WLL), Cordless Phones and broadband access services [2], [3], [4]. This path of rapid evolution has brought with it some of the biggest engineering challenges to date. A couple of years ago, the design of a communication system capable of operating in multiple propagation environments, delivering a broad range of services each with its own quality requirements on a global scale to a person with a single handheld terminal [4] would have sounded far fetched. Today this is reality. The definition of a Universal Mobile Telephone System (UMTS) is currently receiving a great deal of attention from standardization bodies such as the ITU, and current evolution paths predict the availability of UMTS services early in the next century. This process have been the driving force behind the design of new coding, modulation, planning and access methodologies.

Specifically, more dimensions are added to the cellular engineering problem. Starting with FDMA systems, cellular networks have seen the introduction of TDMA systems, CDMA systems and Space Division Multiple Access (SDMA) systems. SDMA systems normally uses one of the other multiple access schemes mentioned, and are realised by adaptive or fixed antenna arrays. An adaptive antenna array for use in a SDMA environment consists of an array of spatially distributed radiating elements, possibly seperated by less than the coherence bandwidth of the channel, where the outputs of each element is adaptively weighted and combined with the other outputs to extract a specific signal from the superposition of signals received.

In our analysis, we will consider a Uniform Linear Array (ULA), where our definition of a uniform array is one of identical elements with identical excitation amplitudes, each with progressive phase. It will further be assumed that the array has maximum radiation directed normal to the axis of the array, making it a broadside radiating array.

The total field of an array is determined by the vector addition of the fields radiated by the individual elements of the array. To create arbitrary directive patterns, it is necessary that the fields from the individual elements of the array interfere constructively in the desired directions, and destructively in the remaining directions. In an array of identical radiating elements, four main variables can be used to control the shape of the overall pattern of the antenna [5]. These are:

1. The array geometry (linear, circular etc.).
2. The excitation amplitude of individual elements.
3. The excitation phase of individual elements.
4. The relative displacement between elements.

Since we assume a ULA in this analysis, items 1. - 3. are known and we therefore only need to determine an optimal displacement between elements to maximise system performance. Using this information, the system Bit Error Rate (BER) in a multipath Rayleigh fading environment is derived in section II, while a correlation parameter to determine the optimal antenna spacing is defined in section III.

In section IV a pdf is proposed to model the non-uniform distribution of mobile users in a cellular environment. A uniform distribution is a special case of the pdf. This is very usefull to model for instance a street crossing where the majority of mobile users will be on the road and therefore non-uniformly distributed through the cell.

The influence of the mobile user location, the number of antenna elements in the array, the number of multipaths and the influence of power control fluctuations on the system performance are considered in section V, while the paper is concluded in section VI.

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II. BER Performance Calculation

Figure 1 is a representation of the cellular system under discussion, with K subscribers in the reference cell. The ouput of the transmitter for user k can then be written as

$$s_k(t) = A_k b_k(t) a_k(t) \cos(\omega_c t + \theta_k) \tag{1}$$

where A_k denotes the transmitted signal amplitude, $b_k(t)$ denotes binary data with bit period T seconds, $a_k(t)$ denotes a random binary spreading sequences with chip period T_c seconds and length $N = T/T_c$. Also, standard Binary Phase Shift Keyeing (BPSK) modulation is used with a carrier frequency of ω_c rad/s and unknown carrier phase θ_k, a random variable uniformly distributes over $[0, 2\pi)$. The transmitted signal propagates over a radio channel modeled as a Rayleigh fading, time invariant, discrete multipath channel with equivalent low-pass response

$$h^{(k)}(\tau) = \sum_{l=1}^{L} \beta_l^{(k)} e^{j\varphi_l^{(k)}} \delta[\tau - \tau_l^{(k)}]. \tag{2}$$

Each path is characterized by the variables $\beta_l^{(k)}$, a Rayleigh distributed random variable denoting the strenght of path l from user k, $\varphi_l^{(k)}$ uniformly distributed over $[0, 2\pi)$ and denoting the phase shift of path l from user k and $\tau_l^{(k)}$, uniformly distributed over $[0, T)$ and denoting the propagation delay of path l from user k. Assuming that $L = \left\lfloor \frac{T}{T_c} \right\rfloor$ multipath components are present, the received signal can be written as

$$\begin{aligned} r(t) \;=\; & \sum_{k=1}^{K} \sum_{l=1}^{L} A_k b_k(t) a_k(t) \beta_l^{(k)} \cdot \\ & \cos(\omega_c t - \omega_c \tau_l^{(k)} + \theta_k + \varphi_l^{(k)}) + \eta(t) \end{aligned} \tag{3}$$

where $\eta(t)$ denotes AWGN with a two-sided power spectral density of $N_0 T/4$.

Since coherent demodulation is assumed, the receiver coherently recovers the carrier phase and delay locks to the desired signal. Assuming that this signal is that of subscriber $k = 1$, the output of the receiver after correlation and demodulation can be written as

$$\begin{aligned} \zeta \;&=\; \mathrm{Re}\left\{ \int_0^T \frac{\bar{w}_1^H}{\|\bar{w}_1\|} r(t) a_1(t) \cos(\omega_c t) dt \right\} \\ &=\; \|\bar{w}_1\| A_k \frac{T}{2} b_0^{(1)} \beta_1^{(1)} \\ &+\; \sum_{l=2}^{L} \frac{A_1}{2} \beta_l^{(1)} \cos(\theta_l^{(1)}) \|\bar{w}_1\| \bar{\mathcal{R}}_{11} \xi_1 \\ &+\; \sum_{k=2}^{K} \sum_{l=1}^{L} \frac{A_k}{2} \beta_l^{(k)} \cos(\theta_l^{(k)}) \|\bar{w}_k\| \bar{\mathcal{R}}_{k1} \xi_k + \eta(t) \end{aligned} \tag{4}$$

where θ is a random variable uniformly distributed on $[0, 2\pi)$,

$$\xi_1 \;=\; \left\{ b_{-1}^{(1)} R_{11}(\tau_l^{(1)}) + b_0^{(1)} \hat{R}_{11}(\tau_l^{(1)}) \right\} \tag{5}$$

$$\xi_k \;=\; \left\{ b_{-1}^{(k)} R_{k1}(\tau_l^{(k)}) + b_0^{(k)} \hat{R}_{k1}(\tau_l^{(k)}) \right\} \tag{6}$$

and

$$\bar{\mathcal{R}}_{k1} = \frac{\mathrm{Re}[\bar{w}_1^H \bar{w}_k]}{\|\bar{w}_1\| \|\bar{w}_k\|} \tag{7}$$

with $(\cdot)^H$ denoting the Hermitian transpose and $\bar{w}_k$ the array manifold vector or steering vector optimising the response of the antenna array for user k. Furthermore, $b_0^{(1)}$ denotes current information bit detected for user $k = 1$ and $b_{-1}^{(1)}$ denotes the previous information bit. Also,

$$\begin{aligned} R_{k1}(\tau_l^{(k)}) \;&=\; \int_0^{\tau_l^{(k)}} a_k(t - \tau_l^{(k)}) a_1(t) dt \\ \hat{R}_{k1}(\tau_l^{(k)}) \;&=\; \int_{\tau_l^{(k)}}^{T} a_k(t - \tau_l^{(k)}) a_1(t) dt. \end{aligned} \tag{8}$$

It should be clear that the detection variable is made up of four distinct elements, each represented by a term in (4). The first term represents the contribution of the main received path of the reference user, the second term all multipath components of this user or self-interference, the third term all transmissions from interfering users within the reference cell and the final term the effect of AWGN. Therefore, (4) can be rewritten as the sum of the desired signal and some interference term, i.e.

$$\zeta = \frac{A_1 T}{2} \left\{ \beta_1^{(1)} \|\bar{w}_1\| b_0^{(1)} + \alpha \right\} + \eta \tag{9}$$

where

$$\begin{aligned} \alpha \;=\;& \sum_{l=2}^{L} \frac{\beta_l^{(1)}}{T} \cos(\theta_l^{(1)}) \|\bar{w}_1\| \bar{\mathcal{R}}_{11} \xi_1 \\ +\;& \sum_{k=2}^{K} \sum_{l=1}^{L} \frac{A_k \beta_l^{(k)}}{A_1 T} \cos(\theta_l^{(k)}) \|\bar{w}_k\| \bar{\mathcal{R}}_{k1} \xi_k \end{aligned} \tag{10}$$

To calculate the error performance of the system, and making use of the Gaussian assumption, that is assuming that α is Gaussian distributed, it is then necessary to calculate the variance of α. In order to simplify the analysis it is assumed that β, θ and τ are all independent random variables. Furthermore, from inspection of (10) it is clear that the expected value of α, $\mathrm{E}\{\alpha\}$ must be equal to zero as $\mathrm{E}\{\cos(\theta)\} = 0$. Therefore, the variance of α is equal to $\mathrm{E}\{\alpha^2\}$ or,

$$\begin{aligned} \mathrm{E}\{\alpha^2\} \;=\;& \sum_{l=2}^{L} \frac{\mathrm{E}\{{\beta_l^{(1)}}^2\}}{\mathrm{E}\{T^2\}} \mathrm{E}\{\|\bar{w}_1\|^2\} \mathrm{E}\{\bar{\mathcal{R}}_{11}^2\} \cdot \\ & \mathrm{E}\left\{ \left(\xi_1 \cos(\theta_l^{(1)}) \right)^2 \right\} \end{aligned}$$

$$
\begin{aligned}
+ \quad & \sum_{k=2}^{K}\sum_{l=1}^{L}\frac{\mathrm{E}\{A_k^2\}\mathrm{E}\{{\beta_l^{(k)}}^2\}}{\mathrm{E}\{A_1^2\}\mathrm{E}\{T^2\}}\mathrm{E}\{\|\bar{w}_k\|^2\}\mathrm{E}\{\bar{\mathcal{R}}_{k1}^2\}\cdot \\
& \mathrm{E}\left\{\left(\xi_k\cos^2(\theta_l^{(k)})\right)^2\right\} \qquad (11)
\end{aligned}
$$

Making use of the results in [6], we can calculate the average error rate in Rayleigh fading. The result is stated in (12).

$$
P_e = \frac{1}{2}\left(1-\sqrt{\frac{\Lambda\Omega_l^{(k)}}{1+\Lambda\Omega_l^{(k)}}}\right) \qquad (12)
$$

where

$$
\frac{1}{\Lambda} = 1 + 2\frac{E_b}{N_0}E\{\alpha^2\}, \qquad (13)
$$

$\Omega_l^{(k)}$ denotes the average strength of path l from user k and $\frac{E_b}{N_0}$ is the energy per bit to noise ratio.

III. OPTIMUM ANTENNA SPACING CRITERIA

In this section we derive a parameter for the optimum antenna spacing from (11) in a cellular CDMA environment. It is clear that if $E\{\alpha^2\} = 0$ the system will have optimal performance, i.e. no multiple access interference. This can only happen when there is no multiple access interference or when $\left(\mathrm{Re}\left[\bar{w}_1^H\bar{w}_k\right]\right)^2 = 0$, which is essentially the spatial correlation between $\phi^{(1)}$ and $\phi^{(k)}$. This is only possible when the interferers, relative to the reference user, are in the antenna nulls. Since we are considering a multiple access system, we would like to optimise $E\{\alpha^2\}$ in the presence of multiple acces interference. The only parameter which can be optimised for this purpose when a ULA is assumed, is to determine an optimal antenna spacing. In order to achieve this, we define the following.

It is well known that for a broadside ULA, there is always a maximum gain at 90° [5]. We therefore assume that the desired user is at a spatially optimal position, $\phi^{(1)} = 90°$. This is arbitrary since it is assumed that, in the case of an adaptive antenna, the main beam of the antenna will always be on the desired user. The other users can be distributed anywhere in the cell. For our derivation of the optimal antenna spacing, we assume the mobile users to be uniformly distributed, that is, the pdf of $\phi^{(k)}$ can be written as

$$
p(\phi^{(k)}) = \frac{1}{2\pi} \qquad \forall \qquad 0 \leq \phi^{(k)} \leq 2\pi. \qquad (14)
$$

We can therefore determine

$$
\begin{aligned}
\Omega &= E\left\{\left(\mathrm{Re}\left[\bar{w}_1^H\bar{w}_k\right]\right)^2\right\} \\
&= E\left\{\left(\mathrm{Re}\left[\sum_{m=0}^{M-1} e^{-j\gamma m d_x\cos(\pi/2)}e^{j\gamma m d_x\cos\phi^{(k)}}\right]\right)^2\right\}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{1}{2\pi}\int_0^{2\pi}\left(\mathrm{Re}\left[\sum_{m=0}^{M-1} e^{j\gamma m d_x\cos\phi^{(k)}}\right]\right)^2 d\phi^{(k)} \\
&= \frac{1}{2}\sum_{x1=0}^{M-1}\sum_{x2=0}^{M-1}\left[J_0(\epsilon(x_1+x_2)) + J_0(\epsilon(x_1-x_2))\right] \qquad (15)
\end{aligned}
$$

where J_0 is the zeroth order Bessel function and

$$
\epsilon = \gamma d_x = \frac{2\pi}{\lambda}\frac{\lambda}{d} = \frac{2\pi}{d} \qquad (16)
$$

where $d \geq 1$ is an integer. Figure 2 shows Ω as a function of d for different values of M.

It is clear that for a given number of antenna elements M, the optimum value of $d = 2$. Therefore an optimum antenna element spacing for a ULA is $d_x = \frac{\lambda}{2}$ and will be used in the performance calculations of section V.

IV. MOBILE LOCATION DISTRIBUTION

The position of mobiles in a cellular system has a significant influence on the system performance. To describe this effect, the cellular multiple access system is assumed to be constituted by a number of non-overlapping cells, each of radius R (in our analysis we only consider one cell). Each cell has a base station located at its center, with mobile users distributed throughout the cell. The position of a mobile user in the cellular structure is fully defined by its distance from the reference base station r, and its angle, ϕ_o, measured from some reference, both of which can be considered random variables. This is shown in Figure 3. As the performance of the cellular system depends on the spatial distribution of mobiles, true pdf's for both the angular and distance distributions of mobiles are required in order to calculate the performance of the system. These are, however, not available in the literature and various approximations such as uniform angular and distance distributions [7] and modified Gaussian distributions [8] are used. These assumptions are either not a good representation of the real world scenario or are somewhat inflexible as it is clear that these distributions will vary from situation to situation. Specifically, a single pdf applicable to many scenarios would be extremely useful. Thus, in terms of the angular distribution of mobiles the authors propose a pdf of the form

$$
\begin{aligned}
p_{\Phi_o}(\phi_o) &= \frac{1}{A_{norm}}\Bigg[1+\sum_{l=1}^{N_{peak}}\gamma_l\Bigg[\mathrm{rect}\left(\frac{w_l\phi_o}{\pi}-\alpha_l\right) \\
&+ \mathrm{rect}\left(\frac{w_l\phi_o}{\pi}-\alpha_l-2\pi\right)\Bigg] \\
&\cdot \cos^2(w_l\phi_o-\alpha_l)\Big] \qquad 0 \leq \phi_o \leq 2\pi \qquad (17)
\end{aligned}
$$

where A_{norm} is a normalizing factor to ensure that $\int_0^{2\pi} p_{\Phi_o}(\phi_o)d\phi_o = 1$ and N_{peak} is the number of peaks in the pdf. This factor (N_{peak}) is a measure of the angular clustering of mobiles in a cell. Clearly, if $N_{peak} = 0$, (17) denotes a uniform angular subscriber distribution. Furthermore, rect(x) is defined as

$$\text{rect}(x) = \begin{cases} 1 & \mid x \mid < \frac{1}{2} \\ 0 & \mid x \mid > \frac{1}{2} \end{cases} \tag{18}$$

with w_l an integer controlling the width of peak l. Typically, values for w_l will be chosen to yield angular peaks of different maximum and minimum widths. For instance, if $w_l = 10$ the width of peak l in the pdf of the angular distribution of mobiles will be $\pi/10$ rad. The angular location of peak l is given by α_l, whilst the relative size of each peak is determined by γ_l (typically values between 0 and 1).

In our BER calculations, the pdf of (17) is used to generate the $\phi^{(k)}$ values, using a procedure in [9] to generate variables of arbitrary distributions.

Figure 4 shows the ULA radiating pattern for $M = 3$ with the reference user located at 90°, while the interfering mobiles are clustered, according to (17), in one of the zeros of the antenna pattern. For these graphs the total number of users $K = 50$. For a antenna element spacing of $d = \lambda/2$, the zeros in the radiation pattern are located at

$$\vartheta_n = \cos^{-1}(\pm\frac{2n}{M}) \qquad \forall \qquad n = 1, 2, 3, \cdots \tag{19}$$

When $M = 10$, it is clear that the zeros located at 0° and 180° are wider than the other zeros. When interfering users are located here, the toal inference would be less and a system performance improvement will be achieved. These type of improvements will be quantified in the next section as a function of the average error rate.

V. RESULTS

In Figure 5 the influence of mobile location is illustrated. The reference user was assumed to be at $\phi^{(1)} = 90°$, while the interfering users were moved to fixed locations as indicated on the graph. Note that the interferers were not distributed through the cell according to (17), but were fixed to the positions indicated. When the interfering mobiles (interfering relative to the reference user) are moved to a null position in the antenna radiation pattern (according to (19)), the BER performance is the same as for a single user in a Rayleigh fading environment. When the interfering mobiles are all moved to the same location as the reference user, $\phi^{(k)} = 90°$, the BER performance is unacceptably high. This is expected since the performance of a CDMA system in a fading environment is unacceptably high. From Figure 4, it is clear that the gain at 180° is smaller than at 90°. By placing the interfering mobiles at $\phi^{(k)} = 180°$, the smaller interference is reflected in the BER curve of Figure 5. By placing the interferers at two arbitrary angles $\phi^{(k)} = 75°$ and $\phi^{(k)} = 30°$, a feeling of the influence of the antenna's radiating pattern can be gained.

Distributing the mobiles according to (17) with interfering mobiles at different peak locations, Figure 6 results. As reference fixed mobile interferers were located at the ULA zero. Since we are generating a pdf with a finite number of users (in this case $K = 30$), the average error rate has to be derived from a number of trials. In this case, and also in all other performance results using (17), thirty trial runs were performed to determine the average error rate. Again the reference user were assumed to be located at $\phi^{(1)} = 90°$, with α_l as indicated in the figure. As can be expected, the performance with $\alpha_l = 0°$ is the best since the antenna null at 0° is the biggest. The performance is still not acceptable and some form of coding or diversity should be implemented to improve the performance to an acceptable level. The system performance for $\alpha_l = 90°$, $\alpha_l = 75°$ and $\alpha_l = 66.42°$ degrees are equally unacceptable, although the performance for $\alpha_l = 66.42°$ (in one of the narrower antenna nulls) is slightly better.

Next we consider the influence of the number of antenna elements on the system performance as depicted in Figure 7. To make a fair comparison, the mobile location peak α_l was chosen such that it is located at one of the antenna array nulls. It is clear that the number of antennas substatntially improve the system performance. As reference, the performance with $M = 1$ and $K = 30$ is compared to different values of M. It is clear from Figure 7 that a large number of antennas are needed to improve the system performance to an acceptable level.

Figure 8 indicated the influence of multipath on the system error performance. Again antenna arrays improve the system performance, but with a realistic number of mulipath components, i.e. $L = 3$, the performance with ten antenna elements are not acceptable. A very large number of antenna elements are needed to improve the system perofrmance in a multipath Rayleigh fading environment or additional techniques such as a RAKE receiver or error control coding is needed.

VI. CONCLUSIONS

In this paper the performance of a multipath Rayleigh faded single cell SDMA/CDMA system was investigated. A criteria for the optimum antenna spacing in a cellular CDMA system employing a ULA was derived. The derivation is based on reducing the multiple access interference by using the spatial correlation as criteria. This criteria can also be used to determine the optimum antenna geometry in a cellular CDMA system. A novel mobile subscriber probability density function was derived to more realistically calculate system performance.

From the results it is clear that antenna arrays are an effective way to reduce interference and to increase system capacity. What is of essential importance is to have an algorithm to determine the angle of arrival of the reference user and to be able to determine where the strongest interferers are. In general, it is clear that, in addition to an antenna array, a RAKE receiver in a multipath environment is essential for acceptable system performance. It should be noted that the antenna arrays have the ability to increase the system capacity substantially, but additional gain can be achieved from a combination with other antenna diversity schemes.

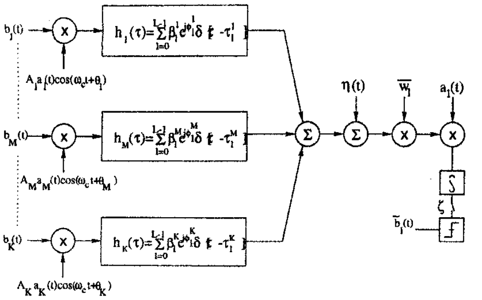

Fig. 1. Basic block diagram of cellular CDMA system.

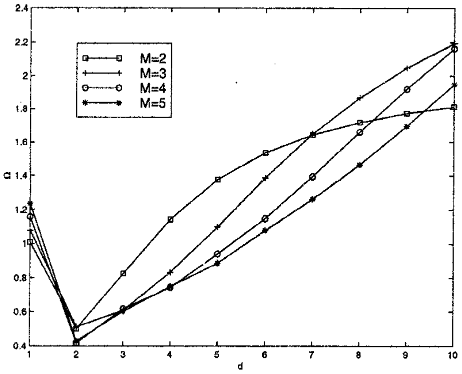

Fig. 2. Radiation pattern for $M = 3$ with mobile users distributed randomly in cell

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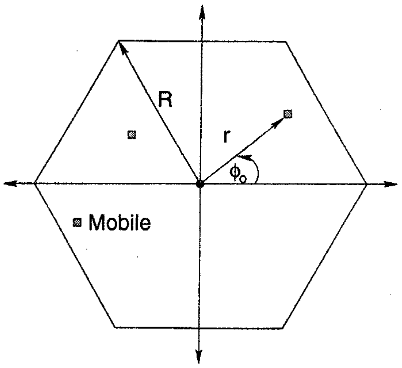

Fig. 3. Modeling the location of mobiles in a cellular system.

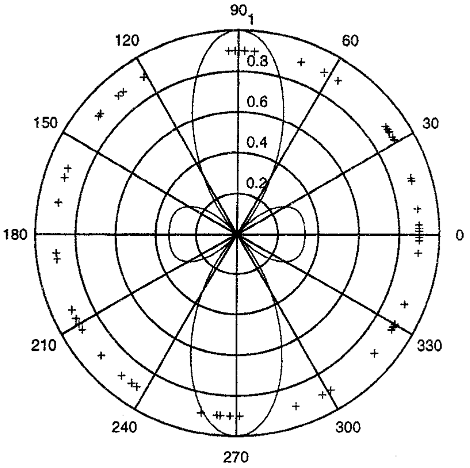

Fig. 4. Radiation pattern of a LNA with $M = 3$ and $\alpha_l = 48.1°$ in (17)

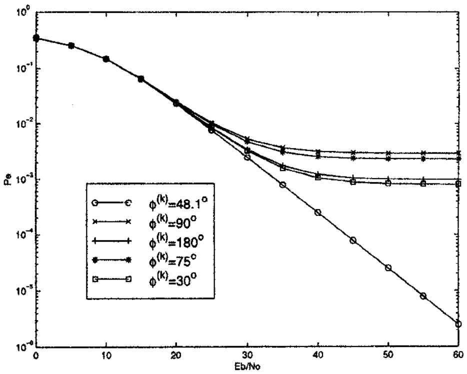

Fig. 5. BER for $K = 10$, $M = 3$, $L = 1$ and $\phi^{(1)} = 90°$

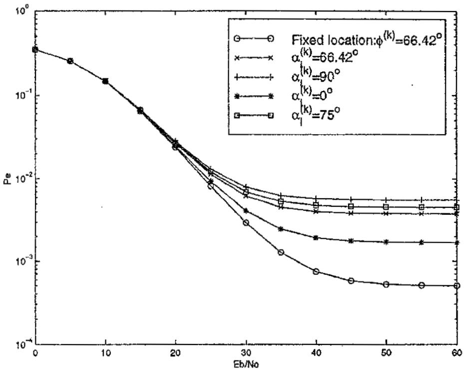

Fig. 6. BER for $K = 30$, $M = 10$, $L = 2$ and $\phi^{(1)} = 90°$

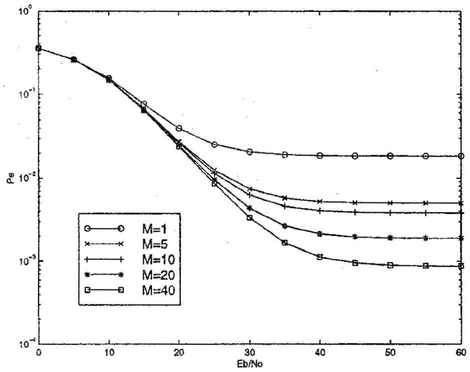

Fig. 7. BER for $K = 30$, $L = 2$, $\phi^{(1)} = 90°$ and α_l at array null

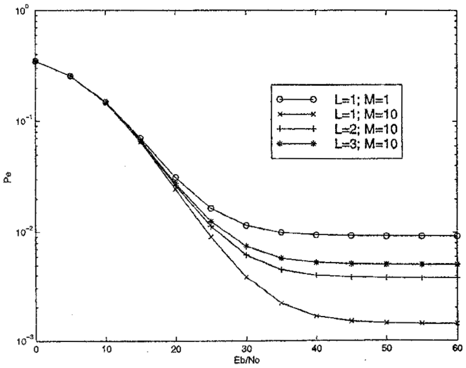

Fig. 8. BER for $K = 30$ and $\phi^{(1)} = 90°$