PERFORMANCE OF CELLULAR SSMA WITH BLOCK CODING UNDER FADING AND MULTIPATH CONDITIONS

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ABSTRACT

In this work we consider the application of channel coding to cellular DS-SSMA digital radio. The method of moments is used to accurately assess the system error probability. Using this technique, we also assess the accuracy of assuming that the multiuser interference has a Gaussian distribution, which allows calculations to be analyzed by simpler means. It is verified that the Gaussian assumption is sufficiently accurate under the conditions considered. Cellular system performance is further investigated with voice activity monitoring and cell splitting under AWGN, Rayleigh fading and multipath conditions typical for personal and mobile communications. Block codes of relative low complexity are investigated when Gold sequences of length 255 and 127 are employed for purposes of PN spreading. The average degradation due to interuser interference is determined by employing exact aperiodic correlation parameters as defined by Pursley [1]. Numerical results indicate that for a given processing gain and number of interfering users, appropriate coding can allow for reliable communication even under multipath fading conditions.

1. INTRODUCTION

Spread spectrum signalling techniques, with its inherent anti- multipath, multiple access and rejection of interference capabilities have increasingly received attention for cellular personal and mobile communications. Until recently the standard analysis of SSMA systems was rather pessimistic about the capacity of these systems compared to FDMA and TDMA. Gilhousen et al [2] recognized that since SSMA capacity is only interference limited (unlike FDMA and TDMA) any reduction in interference converts directly and linearly into an increase in capacity. Therefore, by employing a voice activity factor, sectorizing the cells and using various forms of diversity it is possible to achieve SSMA system capacity at least as good as FDMA and TDMA. This improvement has been indicated by [3] and others under AWGN conditions.

In this paper we give a detailed analysis of the performance of a block coded cellular SSMA system under frequency-selective slowly fading Rayleigh and multipath conditions which is typical to the indoor wireless channel. For benchmark purposes we present the achievement of a coded SSMA system under AWGN conditions.

There is a sizable literature relating to the effects of multiple access interference on the performance of cellular DS-SSMA, among which are [4] and [5]. Yung [4] considered a cellular SSMA system under Rayleigh fading conditions, modelling the multiuser interference as Gaussian noise (as formulated by Pursley [1]). This assumption has been shown to be inaccurate by as much as 20% under certain conditions [5], [6]. The Gaussian assumption, however, simplifies calculations considerably and hence our attempt to proof (or disprove) this conjecture; we use the method of moments, which has been proven to deliver very accurate results [7], to compare average error probabilities to that obtained by the Gaussian assumption. This is accomplished by a family of balanced Gold sequences, of length 255 and 127, with correlation parameter as defined by Sarwarte and Pursley [8].

Using the equations derived in this work it is possible to predict DS/SSMA capacity under the mentioned conditions. We do not, however, set out to predict system capacity, but rather give some comparative results and show that simple block coding can allow for reliable communication under fading conditions. By introducing a voice activity factor of 3/8 and cell splitting of 3, the performance of a 7 cell system with block coding are assessed. We assume hard decisions are made by the demodulator and that the error-producing mechanism results in independent error events or that fading would not cause more than t errors in a block of n bits. The latter assumption requires interleaving at the transmitter and de-interleaving at the receiver.

Section 2 analyses and describes a system model for a typical indoor wireless communication channel. The analyses allows us to calculate average error probabilities by means of the method of moments and the simpler Gaussian assumption. Numerical results are discussed in Section 3. Finally, a summary and conclusions are presented in the last section of the paper.

2. MODEL AND ANALYSIS

The model considered will be summarized briefly and is based on the model developed by in Kavehrad [5].

Measurements by Qualcomm [9] indicate that the adjacent tier in a cellular system contribute approximately 6% per cell of the total interference (for equally loaded cells). Assuming equally loaded cells, the equivalent number of users of the cellular system can be expressed as

$$K = K' \frac{N_{sect}}{V_{on}}(1 + 0.06P_1) \tag{1}$$

where K' is the total number of users per cell, V_{on} the voice activity factor, N_{sect} the cell splitting factor and P_1 the number of interfering cells in the first tier.

Our conjectural system will presume $\frac{N_{sect}}{V_{on}} = 8$, $K' = 10$ and $P_1 = 6$ for a total of approximately 110 users supported by the cellular system.

An equivalent spread spectrum multipath system model for K users is indicated in Figure 1. The channel for the desired transmitter and receiver ($k = 1$) can be represented by an L- paths Rayleigh fading model where a single transmitted pulse is received via L-paths at the random instant $t_l, l = 1, \cdots, L$. We assume t_l is uniformly distributed over one bit period $(0, T)$ and that each user code sequence has a period of $N = T/T_c$.

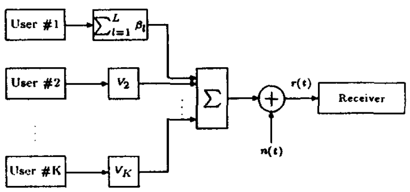

Figure 1: General system architecture

In the analysis we assume that average power control is assumed which also includes averaging the channel fading characteristics. Baseband signalling at a rate less than the channel coherence bandwidth ensures that intersymbol interference can be neglected. Therefore, the channel has a low-pass equivalent impulse response, given by

$$h(t) = \sum_{l=1}^{L} \beta_l \delta(t - t_l) e^{j\phi_l}, \tag{2}$$

where $\delta(\cdot)$ is the delta function, β_l is the Rayleigh distributed path gain and ϕ_l is the random path phase, uniformly distributed between $(0, 2\pi]$.

In the transmission model it is further assumed that the kth interfering user of the multiple access system is linked to the receiver of Figure 1 via a single Rayleigh fading path with a uniformly distributed random delay τ_k ranging from zero to one bit period, T. This will naturally result in a worst case scenario, rendering our results conservative.

In our formulation we specify the Rayleigh distributed path gain of the $K-1$ interfering users by V_k, $k = 2, \cdots, K$. Thus, the received signal for the fading model described is given by

$$\begin{aligned} r(t) &= A \sum_{l=1}^{L} \beta_l a_1(t - t_l) b_1(t - t_l) \cos(\omega_c t + \Phi_l) \quad (3) \\ &+ A \sum_{k=2}^{K} V_k a_k(t - \tau_k) b_k(t - \tau_k) \cos(\omega_c t + \Psi_k) \\ &+ n(t) \end{aligned}$$

where $\Phi_l = \theta_1 - \omega_c t_l + \phi_l$, $\Psi_k = \theta_k - \omega_c \tau_k$ and θ_k the phase of the kth user. Also, $n(t)$ is white Gaussian noise with double sided spectral density of level $N_0/2$ and θ_1 can be assumed zero with no loss of generality. Since coherent PSK is considered, the receiver is assumed to coherently recover the carrier phase and delay lock to the first arriving desired signal. Following Kavehrad [5] and standard procedures, after matched filter reception, the conditional probability of error is given by

$$P_{e|\beta_1,\alpha} = \frac{1}{2} \text{erfc} \left\{ \sqrt{\frac{E_b}{N_0}} [\beta_1 + \alpha] \right\} \tag{4}$$

where

$$\begin{aligned} \alpha &= x + y, \quad (5) \\ x &= \sum_{l=2}^{L} \frac{\beta_l}{T} [b_{-1}^1 R_{1,1}(t_l) + \hat{R}_{1,1}(t_l)] \cos(\Phi_l), \\ y &= \sum_{k=2}^{K} \frac{V_k}{T} [b_{-1}^k R_{k,1}(\tau_k) + b_0^k \hat{R}_{k,1}(\tau_k)] \cos(\Psi_k), \end{aligned}$$

and b_0^1 represents the information bit being detected and b_{-1}^1 is the preceding bit, which, due to the channel delay spread, affects the detection of b_0^1 received on the first path between the desired transmitter and receiver. The parameters β_l and V_k are sample values of a Rayleigh variable. The discrete partial auto- and crosscorrelation functions are given by

$$R_{k,1}(\tau) = A_{n,k,1} T_c + B_{n,k,1}(\tau - nT_c) \tag{6}$$

$$\hat{R}_{k,1}(\tau) = \hat{A}_{n,k,1} T_c + \hat{B}_{n,k,1}(\tau - nT_c) \tag{7}$$

and, together with the variables $A_{n,k,1}, B_{n,k,1}, \hat{A}_{n,k,1}$ and $\hat{B}_{n,k,1}$, defined in [1], enable us to evaluate the system performance for specific code parameters.

The spreading codes used are Gold sequences with generator polynomials 153071 and 41567 (in octal) for $N = 255$ and $N = 127$ respectively. Initial loadings of these codes were chosen in such a way as to generate *balanced* codes (because of their desirable spectral properties) and not necessarily for optimum correlation properties.

Removing the conditioning in β_1 from (4) we have [5]

$$\begin{aligned} P_{e|\alpha} &= \frac{1}{2} \left\{ \text{erfc} \left[-\sqrt{\frac{E_b}{N_0}} \alpha \right] \right. \quad (8) \\ &- \sqrt{\frac{\gamma_0}{\gamma_0 + 1}} \exp \left[\frac{-\frac{E_b}{N_0} \alpha^2}{\gamma_0 + 1} \right] \\ &\left. . \text{erfc} \left[-\sqrt{\frac{E_b}{N_0}} \alpha \sqrt{\frac{\gamma_0}{\gamma_0 + 1}} \right] \right\} \end{aligned}$$

with $\gamma_0 = E\{\beta^2\}\frac{E_b}{N_0}$ and α an approximate Gaussian distributed random variable with zero mean and moments given by equation (34) in Kavehrad [5].

Neglecting fading and multipath (4) reduces to

$$P_{e|y} = \frac{1}{2}\text{erfc}\left\{\sqrt{\frac{E_b}{N_0}}[1+y]\right\} \tag{9}$$

with $V_k = 1$ in (3).

There are two ways to remove the conditioning in (8) and (9). The more accurate way is to employ Gauss Quadrature integration [10] by averaging the conditional probability in (8) and (9) over the interference term α and y respectively. Briefly, this is accomplished by evaluating the moments of α and y, which are applied in evaluation of the weights and nodes of the Quadrature Rule.

The alternative method is to assume α and y to be Gaussian distributed variables with zero mean and variance σ^2_{ma} given by the second moment of (34) in [5]. With this assumption, a closed form expression for (8) and (9) can be derived at as

$$P_e = \frac{1}{2\gamma_0}\sqrt{\Gamma}\left\{\sqrt{\frac{1}{\Gamma}}+\sqrt{\Lambda}\right\}^{-1} \tag{10}$$

and

$$P_e = \frac{1}{2}\text{erfc}\left\{\left(\frac{N_0}{E_b}+2\sigma^2_{ma}\right)^{-\frac{1}{2}}\right\}, \tag{11}$$

respectively, with

$$\Gamma = \frac{\gamma_0(1+\Lambda)}{1+\Lambda(1+\gamma_0)} \tag{12}$$

and

$$\frac{1}{\Lambda} = \frac{E_b}{N_0}2\sigma^2_{ma}. \tag{13}$$

We notice that (11) is similar to an expression derived by Pursley [1] for PSK signalling under AWGN conditions and that in the absence of multiple access interference and a single- path fading of the desired signal (8) and (10) simplifies to

$$P_e = \frac{1}{2}\left\{1-\sqrt{\frac{\gamma_0}{1+\gamma_0}}\right\}, \tag{14}$$

which is the ideal performance of a single-path Rayleigh fading channel [11].

2.1 Channel Coding

Error control coding can be used with great success in SSMA with no penalty paid in bandwidth by the addition of redundancy to the information bits. In this work the $(n,k,t) = (7,4,1)$ Hamming code and five BCH codes are investigated. The BCH codes under investigation are the $(15,7,2), (31,16,3), (63,30,6), (127,64,10)$ and the $(255,123,19)$ codes. These are all approximately rate $R_{cd} = \frac{k}{n} \approx \frac{1}{2}$ codes.

We assume that the PN spreading sequence spans one code symbol. This implies that under the assumption of fixed throughput (i.e. constant data rate), fixed maximum chip rate and fixed complexity, a rate R_{cd} code must employ a PN spreading sequence shorter by a factor R_{cd} than that of the uncoded case. This results in increased interuser interference due to the poorer cross correlation properties of shorter PN sequences. In our case we use PN sequences of $N = 255$ and $N = 127$ for the uncoded case and coded cases respectively.

For a channel code that corrects t-errors, the bit error probability is given as [12]

$$P_b \leq \sum_{i=t+1}^{n} \frac{i+t}{n}\binom{n}{i}P_e^i(1-P_e)^{n-i}, \tag{15}$$

where n is the coded block length and P_e is the average bit error probability of (8), (9), (10) or (11).

3. Numerical results

We start our discussion by comparing the Gaussian assumption with GQR integration. In all calculations concerning Gauss Quadrature integration 19 moments were used. Concluding the section is detailed performance results of three block coded scenarios.

3.1 Accuracy of the Gaussian assumption

Figure 2 depicts the performance of a SSMA system with $K = 110$. Under Rayleigh fading conditions (8) and (10) are used to perform the comparison. It is clear that for this case there is virtually no difference, although the Gaussian assumption approach results in a slightly better error rate than the method of moments for $N = 127$. With block coding the difference becomes even less significant; GQR integration and the Gaussian assumption differ only in the second decimal and the performance is as depicted in Figure 4.

Under AWGN conditions with no fading or multipath, (9) and (11) are used. From Figure 2 it is clear that the difference is also marginal, with the Gaussian assumption yielding slightly lower values of P_e. As expected the assumption becomes more accurate as N increases. With block coding, as in the Rayleigh faded case, the difference is also less significant and as indicated in Figure 3.

We therefore conclude that the Gaussian assumption is valid when block coding is considered.

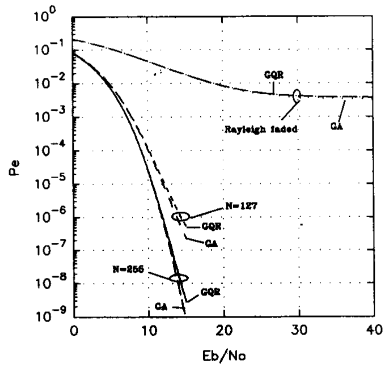

Figure 2: Comparison of P_e calculated with GQR and the Gaussian assumption

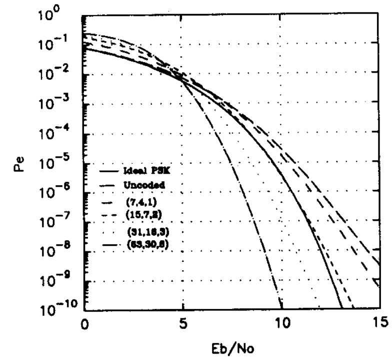

Figure 3: Average error performance for AWGN channel

3.2 Case study

To assess the performance of a cellular SSMA system we consider three separate cases:

Case 1 - All signals from transmitter to receiver are corrupted by AWGN. That is, the variables β_k, V_k and L, as depicted in Figure 1 and (3), are all unity. We use (11) to calculate the average probability of error. This will serve as benchmark to the faded and multipath cases.

Case 2 - Here we consider the case where $L = 1$, thus eliminating multipath effects on the desired user. All the signals arriving via different paths at the receiver have Rayleigh distributed random gains and therefore σ^2_{ma} is calculated for $L = 1$. (For $N = 255$, $L = 1$ and $K' = 10$, $\sigma^2_{ma} = 0.000365535$)

Case 3 - All the *desired* signals arriving via different paths at the receiver together with the interfering signals have Rayleigh distributed random gains. The number of desired paths is limited to ten and σ^2_{ma} in (10) include both fading for the k users and the L paths. This is a scenario in which the transmitter terminals are mobile and gains are Rayleigh with respect to geographic positioning. (For $N = 255$, $L = 10$ and $K' = 10$, $\sigma^2_{ma} = 0.000864594$)

In Cases 2 and 3 all average path gains between the desired transmitter and receiver were assumed to be equal. This assumption will result in conservative error probability values for a fixed total interference power.

3.3 Detailed Results

Figure 3 illustrates the average error probability as a function of unfaded signal-to-noise ratio corresponding to Case 1. In the same figure, performance of an ideal coherent PSK demodulator is shown.

It is clear that with a processing gain of 24 dB ($N = 255$), without coding, a cellular system supporting 110 users can be accommodated quite easily under AWGN conditions. By introducing error control coding the situation is improved even further. It is clear that coding is only beneficial for signal-to-noise values greater than 5 dB.

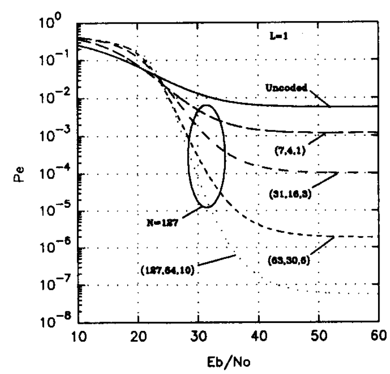

Figure 4: Average error performance under Rayleigh fading

Considering Case 2, where the transmitters are mobile and multipath is neglected, the picture looks a lot worse than that of Figure 2. We assume a hypothetical average path strength of the Rayleigh faded path associated with the kth user to be -15 dB. It is evident that the performance with 110 users is totally unacceptable; the curve saturates at an average error probability of approximately 0.006. By introducing an error control code that can correct three or fewer errors, the BCH $(31, 16, 3)$ code in this case, the average error probability saturates at approximately 10^{-4}, which is already acceptable for speech. More powerful codes improve the situation even further. However, coding is only beneficial at high values of signal-to-noise ratios, typical 22 dB in this scenario. If lower signal-to-noise ratios are required other forms of diversity has to be considered.

Nevertheless, error control alone, without additional forms of diversity, is sufficient to allow for acceptable communication under Rayleigh fading conditions.

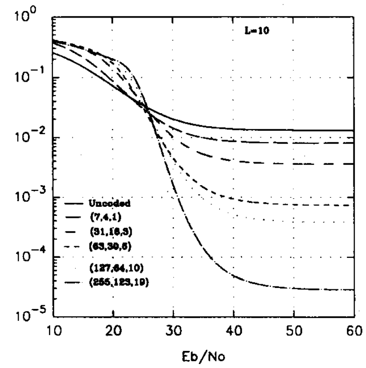

Figure 5: Average error performance under multipath conditions

From Figure 5 we see the atrocious performance of the uncoded signal with ten multipaths. Powerful coding is needed to improve the situation. At least a BCH (127,64,10) is needed to rescue the situation, although the (255,123,19) code would be preferred. As was the case in the previous situations, coding only improve the situation at high signal-to-noise ratios, in this case 27 dB.

It is thus evident that relatively simple error control codes play an important role in cellular SSMA systems.

4. Summary and Conclusions

The work reported extends previous results in the following respects. Accurate closed form expressions were derived using the Gaussian assumption. The performance of a cellular, block coded DS-SSMA was evaluated over Rayleigh and multipath fading channels.
To summarize: our transmission medium is modelled as a discrete number of resolved paths (equal to ten), with each path having a Rayleigh distributed gain. The Gaussian assumption is valid when compared to GQR integration under block coded conditions. Further, block coding is an efficient way to establish communication under fading and multipath conditions.
From our numerical results we draw the following conclusions:
To model the interuser interference as Gaussian noise with block coding is sufficiently accurate under fading and multipath conditions. In an AWGN environment with modest coding, a cellular DS-SSMA system deliver sufficient performance, comparing favourably to TDMA. When Rayleigh fading and multipath conditions are considered without coding, it seems absolutely necessary to include some form of diversity; otherwise, much higher processing gain is needed to decrease the error probability. When simple block codes are used as a form of diversity, the performance is acceptable under these conditions. If better performance is required under fading and multipath conditions, additional forms of diversity and/or more powerful error control codes (with perhaps soft decision) is required.

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