Rake Receiver Performance for CDMA Downlink

Ning Kong, Ian Riphagen, Michiel Lotter, Mark Kent and Pieter van Rooyen
Zyray Wireless Inc. 11455 El Camino Real, Suite 350
San Diego, CA 92130, USA, nkong@zyraywireless.com

Abstract - This paper presents a closed-form expression of the average SIR of the optimum CDMA downlink Rake receiver [1] for identically independently distributed (iid) Raleigh fading and a tight bound for the non-iid case. It shows that the average SIR decreases as the number of multipaths, L, increases and achieves its minimum when the channel multipath intensity profile (MIP) is constant. This paper also proves, both in theory and with simulations that the BER increases as L increases, and that it is a convex function of the multipath channel gains and achieves its maximum when the channel MIP is constant. In addition, this paper derives a decision threshold for MRC, which results in a non-optimum Rake, to determine if a multipath is to be combined without loss of SIR. In summary, this paper proves, against conventional wisdom, that a Rake receiver for the CDMA downlink does not achieve the Lth order diversity gain. In fact, its performance varies inversely to an Lth order diversity system, such as CDMA uplink.

I. Introduction

MRC, where multipath signals are weighted by their complex conjugate of the channel impulse responses, is found to combat only multipath fading not multipath interference (MPI) [1]. The only MPI suppression for a MRC Rake receiver comes from despreading the intended multipath where the MPI is suppressed by a factor equal to the processing gain. In spite of so many proposed interference mitigating receivers [2-7], Rake with MRC has mostly been implemented on CDMA wireless receivers, especially handsets, due to its simplicity, low cost and reliable performance. The higher complexity of the present downlink interference mitigation receivers, such as a chip equalizer either performing zero forcing, MMSE or DFE to restore the orthogonality between multipath signals, lies in the need to calculate the inversion of the channel matrix at the chip rate [2-6]. The adaptive equalizing algorithms such as LMS and RLS help to reduce the complexity, however they do not always converge [7]. G-Rake [8] also has more complexity than MRC. Its matched filter is sampled and correlated at timings in addition to those of the multipaths in order to create correlation among finger outputs and it needs to estimate the interference correlation matrix and calculate its inverse. Keeping the same complexity as a Rake with MRC, [1] introduced an interference mitigating optimum combining (IMOC) scheme, which not only combats fading but also mitigates MPI. In fact, a Rake with IMOC is the optimum Rake. It always performs reliably better than MRC. [1] derived the closed form expression for the instantaneous SINR for the IMOC and proved, by the Schwarz inequality, that it is an upper bound on that of MRC. It was further shown in [1] that the combined SINR for MRC is not a chi square random variable which has a degree of freedom equal to the number of multipaths, as in the case of an uplink or as in [9, Eq.(7.5.16)] assuming the CDMA codes were orthogonal at all shifts thereby no MPI exists when using MRC. Therefore, surprisingly, the concept of MRC for CDMA downlink having a diversity gain of an order equal to the number of the combined multipaths [e.g.,10] is flawed. The combined SINR of IMOC is a sum of the SINR of each finger, which has an F probability density function. It consists of a numerator having a Chi Square random variable (rv) with 2 degrees of freedom from the intended multipath and the denominator having a Chi Square rv with $2L-2$ degrees of freedom from the $L-1$ MPI's.

In this paper, we derive a closed form expression for the average SIR of the IMOC for iid Raleigh fading channels. This average SIR is found to be monotonically decreasing as L increases. For the non-iid Raleigh fading channels, we derive a lower bound for the SIR and this bound is tight when applied to the SIR for the iid case. The bound is also proved to be minimized when the channel MIP is constant. The paper shows both in theory and in simulations that BERs of both the IMOC and MRC obtain their maximum when the channel MIP is constant and increase as L increases. Therefore, the BER of a Rake receiver with either IMOC or MRC in the presence of MPI behaves in an opposite way to the BER with a diversity gain of order L where the BER decreases as L increases and obtains its minimum when the channel MIP is constant.

As shown in [1], the SINR of MRC is not guaranteed to increase as the number of combined multipaths m $(1 < m < L)$ increases. In order to prevent the SINR loss due to combining, we derive the condition when a particular multipath will make the combined SINR of MRC decrease. This condition is determined by two thresholds. The first is a measure of the ratio of the increments in the signal and the interference resulting from combining the multipath. The second is a measure of the multipath channel gain compared to those of other multipaths.

The paper is organized as follows. Section II derives the condition when a particular multipath will make the combined SIR of MRC decrease. Section III derives the average combined SIR of IMOC for both non-iid and iid fading scenarios. It shows the average combined SIR decreases as L increases and for the same number of the multipaths, the bound obtains a minimum when the channel MIP is constant. Section IV analyzes the BERs of the IMOC and proves in theory, consistent with the SIR's reaching its minimum in the iid fading case, that it reaches its maximum at the same time. Section V presents simulation results, which agrees with the theoretical analysis. Finally Section VI concludes.

0-7803-8521-7/04/$20.00 © 2004 IEEE

II. Conditions for the SIR of MRC to increase when combining multipath components

The signal-to-interference-plus-noise ratio of a Rake receiver is

$$SINR = E\left(\left\|\mathbf{w}^H\mathbf{d}\right\|^2\right)\Big/\left[\left\|\mathbf{w}^H(\mathbf{i}+\mathbf{n})\right\|^2\right] = \frac{\mathbf{w}^H(\boldsymbol{\alpha}\mathbf{e}^{\mathbf{j}\boldsymbol{\varphi}})(\boldsymbol{\alpha}\mathbf{e}^{\mathbf{j}\boldsymbol{\varphi}})^H\mathbf{w}NP_s}{\mathbf{w}^H\mathbf{R}_{i+n}\mathbf{w}} \quad (1)$$

where $\mathbf{w}^H = (w_0, w_2, \cdots, w_{L-1})^*$ are the weights assoiated with the L multipath signals to maximize the combined SINR, the superscript indicatess complex conjugate transpose. $\mathbf{d}^T = A_1b_1N(\boldsymbol{\alpha}\mathbf{e}^{\mathbf{j}\boldsymbol{\varphi}})^T = A_1b_1N(\alpha_0e^{j\varphi_0}, \alpha_1e^{j\varphi_1}, \cdots \alpha_{L-1}e^{j\varphi_{L-1}})$ is the desired signal with L components from L multipaths, where b_1 is the desired user information bit, A_1 is its amplitude without channel impairments, N is the processing gain from dispreading, P_S is the desired signal transmitted power, $(\alpha_0e^{j\varphi_0}, \alpha_1e^{j\varphi_1}, \cdots \alpha_{L-1}e^{j\varphi_{L-1}})^T$ is a column vector of the L channel impulse responses and each element is a zero-mean complex Gaussian random variable with a Rayleigh distributed amplitude and uniform distributed angle over $(0, 2\pi]$. $\alpha_i^2, i = 1, 2, \cdots, L$ are L exponential random variables. $\mathbf{R}_{i+n}$ is the interference plus noise correlation matrix and is given by [1],

$$\mathbf{R}_{i+n} = (r_{ij}) = E(i_ii_j) + E(n_in_j)$$

$$= NP_i\text{diag}\left(\sum_{l\neq 0}^{L-1}\alpha_l^2 + N\sigma_n^2 \quad \sum_{l\neq 2}^{L-1}\alpha_l^2 + N\sigma_n^2 \quad \cdots \quad \sum_{l\neq L-1}^{L-1}\alpha_l^2 + N\sigma_n^2\right) \quad (2)$$

where A_k is the kth user's amplitude without channel impairments, $P_i = \sum_{k=0}^{K-1}A_k^2$ is the total interference power without the channel impairments. Note that P_i also includes the desired user's power, as the desired user's signal is interference when coming from another multipath. $\mathbf{R}_{i+n}$ is diagonal due to the fact that correlations among different multipaths are approximately zero since they are at least a chip apart. Making a change of variable, $\mathbf{z} = \mathbf{R}_{i+n}^{1/2}\mathbf{w}$, and using the Schwarz inequality, the optimum weights, denoted by $\mathbf{w}_{IMOC}$, occurs when $\mathbf{z} = \mathbf{R}_{i+n}^{-1/2}\boldsymbol{\alpha}\mathbf{e}^{\mathbf{j}\boldsymbol{\varphi}}$, and $\mathbf{w}_{IOMC} = \mathbf{R}_{i+n}^{-1}\boldsymbol{\alpha}\mathbf{e}^{\mathbf{j}\boldsymbol{\varphi}} =$

$$\text{diag}\left(1/P_i\sum_{l\neq 0}^{L-1}\alpha_l^2 + N\sigma_n^2, \quad 1/P_i\sum_{l\neq 1}^{L-1}\alpha_l^2 + N\sigma_n^2, \quad \cdots \quad 1/P_i\sum_{l\neq L-1}^{L-1}\alpha_l^2 + N\sigma_n^2,\right)\boldsymbol{\alpha}\mathbf{e}^{\mathbf{j}\boldsymbol{\varphi}} \quad (3)$$

Note that $\mathbf{w}_{IMOC}$ becomes $\mathbf{w}_{MRC}$ if there is no multipath component. The optimum SINR resulting from (3), denoted by $SINR_{IMOC}$ is

$$SINR_{IMOCC} = (\boldsymbol{\alpha}\mathbf{e}^{\mathbf{j}\boldsymbol{\varphi}})^H\, diag\left(1/P_i\sum_{l\neq 0}^{L}\alpha_l^2 + N\sigma_n^2, \qquad \cdots \quad P_i\sum_{l\neq L-1}^{L-1}\alpha_l^2 + N\sigma_n^2\right)\boldsymbol{\alpha}\mathbf{e}^{\mathbf{j}\boldsymbol{\varphi}}NP_s$$

$$= \sum_{i=0}^{L-1}\left(\alpha_i^2/P_i\sum_{l\neq i}^{L}\alpha_l^2 + N\sigma_n^2\right)NP_s \quad (4)$$

Compared to the SINR of MRC, denoted by $SINR_{MRC}$ which can be easily derived as

$$SINR_{MRC} = \frac{\left((\boldsymbol{\alpha}\mathbf{e}^{\mathbf{j}\boldsymbol{\varphi}})^H(\boldsymbol{\alpha}\mathbf{e}^{\mathbf{j}\boldsymbol{\varphi}})\right)^2NP_s}{(\boldsymbol{\alpha}\mathbf{e}^{\mathbf{j}\boldsymbol{\varphi}})^H\text{diag}\left(P_i\sum_{l\neq 1}^{L}\alpha_l^2 + N\sigma_n^2 \quad \cdots \quad P_i\sum_{l\neq L}^{L}\alpha_l^2 + N\sigma_n^2\right)(\boldsymbol{\alpha}\mathbf{e}^{\mathbf{j}\boldsymbol{\varphi}})}$$

$$= \frac{\left\|(\boldsymbol{\alpha}\mathbf{e}^{\mathbf{j}\boldsymbol{\varphi}})\right\|^4NP_s}{\sum_{i=1}^{L}\left(P_i\sum_{l\neq i}^{L}\alpha_l^2 + N\sigma_n^2\right)\left|\alpha_ie^{j\varphi_i}\right|^2} \quad (5)$$

[1] has proven that (5) is always upper bounded by (4). If reasonably assuming that a WCDMA system is interference limited [1], then (3) is simplified as

$$\mathbf{w}_{OIMC} = \mathbf{R}_{i+n}^{-1}\boldsymbol{\alpha}\mathbf{e}^{\mathbf{j}\boldsymbol{\varphi}} = diag\left(1/\sum_{l\neq 0}^{L-1}\alpha_l^2, \quad 1/\sum_{l\neq 1}^{L-1}\alpha_l^2 \quad \cdots \quad 1/\sum_{l\neq L-1}^{L-1}\alpha_l^2\right)\boldsymbol{\alpha}\mathbf{e}^{\mathbf{j}\boldsymbol{\varphi}} \quad (6)$$

Consequently, the resulting instantaneous SIR of IMOC and MRC are the following respectively:

$$SIR_{IMOC} = \sum_{i=1}^{L}\frac{\alpha_i^2}{\sum_{l\neq i}^{L}\alpha_l^2}\Gamma_0 \quad (7) \quad \text{and} \quad SIR_{MRC} = \frac{\left(\sum_{i=1}^{L}\alpha_i^2\right)^2}{\sum_{i=1}^{L}\left(\sum_{l\neq i}^{L}\alpha_l^2\right)\alpha_i^2}\Gamma_0 \quad (8)$$

$\Gamma_0 \equiv NP_S/P_I$. Applying the idea of generalized selecting combining scheme [11,12], we combine m, where $m \le L$ strongest multipaths from L total existing multipaths, then its SINR denoted by $SIR_{OC}^{(m/L)}$ for IMOC and $SIR_{MRC}^{(m/L)}$ are

$$SIR_{IMOC}^{(m/L)} = \sum_{i=1}^{m}\frac{\alpha_i^2}{\sum_{l\neq i}^{L}\alpha_l^2}\Gamma_0 \quad \text{and} \quad SIR_{MRC}^{(m/L)} = \frac{\left(\sum_{i=1}^{m}\alpha_i^2\right)^2}{\sum_{i=1}^{m}\left(\sum_{l\neq i}^{L}\alpha_l^2\right)\alpha_i^2}\Gamma_0$$

respectively. Clearly, $SIR_{IMOC}^{(m/L)}$ monotonically increases as m increases. However, $SINR_{MRC}^{(m/L)}$ increases with m conditionally and the condition for $SIR_{MRC}^{(m/L)}$ to decrease as m increases can be found as follows: Let $SIR_{MRC}^{(m+1/L)}$ be the SIR after combining the $m+1$ th multipath, then

$$SIR_{MRC}^{(m+1/L)} = \frac{\left(\sum_{i=1}^{m+1}\alpha_i^2\right)^2}{\sum_{i=1}^{m+1}\left(\sum_{l\neq i}^{L}\alpha_l^2\right)\alpha_i^2}\Gamma_0 \equiv \frac{x_{m+1}}{y_{m+1}}\Gamma_0$$

At the same time,

$$SIR_{MRC}^{(m+1/L)} = \frac{\left(\sum_{i=1}^{m}\alpha_i^2\right)^2 + 2\sum_{i=1}^{m}\alpha_i^2\alpha_{m+1}^2 + \alpha_{m+1}^4}{\sum_{i=1}^{m}\left(\sum_{l\neq i}^{L}\alpha_l^2\right)\alpha_i^2 + \left(\sum_{l\neq i}^{L}\alpha_l^2\right)\alpha_{m+1}^2}\Gamma_0 \equiv \frac{x_m + \Delta x_m}{y_m + \Delta y_m}\Gamma_0$$

where

$x_m \equiv \left(\sum_{i=1}^{m}\alpha_i^2\right)^2$, $y_m \equiv \sum_{i=1}^{m}\left(\sum_{l\neq i}^{L}\alpha_l^2\right)\alpha_i^2$, $\Delta x_m \equiv 2\sum_{i=1}^{m}\alpha_i^2\alpha_{m+1}^2 + \alpha_{m+1}^4$,

$\Delta y_m \equiv \left(\sum_{l\neq m+1}^{L}\alpha_l^2\right)\alpha_{m+1}^2$ and $\frac{x_m}{y_m}\Gamma_0 \equiv SINR_{MRC}^{(m/L)}$. Therefore, if $SINR_{MRC}^{(m+1/L)} = \frac{x_m + \Delta x_m}{y_m + \Delta y_m}\Gamma_0 \ge SINR_{MRC}^{(m/L)} = \frac{x_m}{y_m}\Gamma_0$ then, it can be easily derived that the following needs to be true:

$$\frac{\Delta x_m}{\Delta y_m} \ge \frac{x_m}{y_m}, \text{ or } \frac{2\sum_{i=1}^{m}\alpha_i^2 + \alpha_{m+1}^2}{\left(\sum_{l\neq m+1}^{L}\alpha_l^2\right)} \ge \frac{\left(\sum_{i=1}^{m}\alpha_i^2\right)^2}{\sum_{i=1}^{m}\left(\sum_{l\neq i}^{L}\alpha_l^2\right)\alpha_i^2} \quad (9)$$

and the converse is also true. Hence $SINR_{MRC}^{(m+1/L)} \ge SINR_{MRC}^{(m/L)}$ is true if and only if (9) is true. (9) indicates that, when combining another multipath, if the ratio of the increments of the signal power and the interference power is greater than the SIR before combining, then after combining, the combined SIR will increases, otherwise it will decrease. Since the channel MIP is arbitrary, the increase of the SIR (or SINR) for MRC by combining more MPs is not guaranteed. Alternatively, (9) can be rewritten as

$$\alpha_{m+1}^2 \geq \frac{\left(\sum_{i=1}^{m} \alpha_i^2\right)^2}{\sum_{i=1}^{m}\left(\sum_{l\neq i}^{L} \alpha_l^2\right)\alpha_i^2} - 2\sum_{i=1}^{m} \alpha_i^2 \qquad (10)$$

which provides a threshold , using the power of the next multipath to prevent the SIR loss due to combining.

III. SIR analysis of and IMOC for both iid and non-iid fading MP channels

The average SIR, from (7) for an iid multipath fading is

$$\Gamma_{IMOC\_iid}^{(L/L)} = E\left(SINR_{IMOC}^{(L/L)}\right) = E\left(\sum_{i=1}^{L} \frac{\alpha_i^2}{\sum_{l\neq i}^{L} \alpha_l^2}\right)\Gamma_0 = \sum_{i=1}^{L} E\left(\frac{\alpha_i^2}{\sum_{l\neq i}^{L} \alpha_l^2}\right)\Gamma_0$$

$$= \sum_{i=1}^{L} E\left(\alpha_i^2\right)E\left(\frac{1}{\sum_{l\neq i}^{L} \alpha_l^2}\right)\Gamma_0 = \sum_{i=1}^{L} \frac{1}{L-2}\Gamma_0 = \frac{L}{L-2}\Gamma_0 \qquad (11)$$

where the fourth equality comes from the independence of the numerator and denominator. And the fifth quality comes from $E\left(\alpha_i^2\right) = 1/c$ and $E\left(1/\sum_{l\neq i}^{L} \alpha_l^2\right) = c/(L-2)$.

Since $\Gamma_{IMOC\_iid}^{(L/L)} = \frac{L}{L-2}\Gamma_0 = \Gamma_0 + \frac{2}{L-2}\Gamma_0$, clearly $\Gamma_{IMOC\_iid}^{(L/L)}$ monotonically decreases as L increases. For non-iid Raleigh fading channels,

$$\Gamma_{IMOC\_non\_iid}^{(L/L)} = E\left(SINR_{IMOC\_non\_iid}^{(L/L)}\right) = E\left(\sum_{i=1}^{L}\alpha_i^2 \Big/ \sum_{l\neq i}^{L} \alpha_l^2\right)\Gamma_0 =$$

$$\sum_{i=1}^{L} E\left(\alpha_i^2\right)E\left(\frac{1}{\sum_{l\neq i}^{L} \alpha_l^2}\right)\Gamma_0 \geq \sum_{i=1}^{L} \frac{1}{c_i}\frac{1}{\sum_{l\neq i}^{L} E\left(\alpha_l^2\right)}\Gamma_0 = \sum_{i=1}^{L} \frac{1}{c_i}\frac{1}{\sum_{l\neq i}^{L} \frac{1}{c_l}}\Gamma_0$$

where the inequality comes from the convexity of the function and application of the Jesen's inequality. Note that if $c_i = c_j$, for $i \neq j$ then $\Gamma_{IMOC\_non\_iid}^{(L/L)} = \Gamma_{IMOC\_iid}^{(L/L)} > \frac{L}{L-1}\Gamma_0$. Comparing to (5), the bound is obviously tight. If we use this bound as an approximation to $\Gamma_{IMOC\_non\_iid}^{(L/L)}$, then

$\Gamma_{IMOC\_non\_iid}^{(L/L)} \approx \sum_{i=1}^{L} \frac{1}{c_i}\frac{1}{\sum_{l\neq i}^{L} 1/c_l}\Gamma_0 = \sum_{i=1}^{L} \frac{1/c_i}{1-1/c_i}\Gamma_0$. Then we can

prove that $\Gamma_{IMOC\_non\_iid}^{(L/L)}$ achieves its minimum when the channel MIP is constant. Using Schwartz's inequality,

$$\Gamma_{IMOC\_non\_iid}^{(L/L)} = -L\Gamma_0 + \frac{1}{L-1}[(1-\frac{1}{c_1}) + (1-\frac{1}{c_2}) + \cdots + (1-\frac{1}{c_L})]$$

$$\times(\frac{1}{1-\frac{1}{c_1}} + \frac{1}{1-\frac{1}{c_2}} + \cdots + \frac{1}{1-\frac{1}{c_L}})\Gamma_0 \geq -L\Gamma_0 + \frac{\Gamma_0}{L-1}$$

$$\times(\sqrt{1-\frac{1}{c_1}}\frac{1}{\sqrt{1-\frac{1}{c_1}}} + \sqrt{1-\frac{1}{c_2}}\frac{1}{\sqrt{1-\frac{1}{c_2}}} + \cdots + \sqrt{1-\frac{1}{c_L}})^2 = \frac{L}{L-1}\Gamma_0$$

The equality holds when $c_i = c_j$, for $i \neq j$.

III. The average BER of IMOC

The average BER of IMOC for non iid multipath fading, denoted by P_b can be expressed as the following:

$$P_b = E\left(Q(\sqrt{2\Gamma_{IMOC\_non\_iid}^{(L/L)}})\right) = E\left(Q(\sqrt{2\sum_{i=1}^{L} z_i/(\sum_{l\neq i}^{L} z_l)\Gamma_0}\right)$$

$$= \int_0^{\infty}\cdots\int_0^{\infty} Q\left(\sqrt{2\sum_{i=1}^{L} z_i \Big/ \left(\sum_{l\neq i}^{L} z_l\right)\Gamma_0}\right) d\phi \prod_{j=1}^{L} c_j e^{-(c_1z_1+c_2z_2+\cdots+c_Lz_L)} dz_1 dz_2 \cdots dz_L$$

$$= \int_0^{\infty}\cdots\int_0^{\infty} \frac{1}{\pi}\int_0^{\frac{\pi}{2}} e^{-\frac{1}{(\sin^2\phi)}\sum_{i=1}^{L}\frac{z_i}{\left(\sum_{l\neq i}^{L} z_l\right)}\Gamma_0} d\phi \prod_{j=1}^{L} c_j e^{-(c_1z_1+c_2z_2+\cdots+c_Lz_L)} dz_1 dz_2 \cdots dz_L \quad (12)$$

Note that the fourth equality follows from the alternate expression of the Q-function [12]. It is difficult to obtain a closed form expression of the average BER. However, we prove in Appendix A, that the average BER of the downlink IMOC has a maximum, under the constraint of $\sum_{j=1}^{L} 1/c_j = a$, when the channel MIP is constant. This is opposite to BER property with a diversity gain of an order L. Since the BER having a diversity gain of order L satisfies: $P_b^{(L^{th}\text{ diversity order})} \approx 1/(\prod_{j=1}^{L} 1/c_j)$ [9], and by the inequality of the geometric mean being always less or equal to the arithmetical mean, $\prod_{j=1}^{L} \frac{1}{c_j} \leq \left(\frac{1}{L}\sum_{j=1}^{L}\frac{1}{c_j}\right)^L = \left(\frac{a}{L}\right)^L$ and the equality holds if and only if $c_i = c_j$, for $i \neq j$. Therefore, $P_b^{(L^{th}\text{ diversity order})}$ obtains it minimum when the channel has a constant MIP. It is intuitively true since diversity performs best, when each multipath has equal power.

IV. Simulation Results

In this Section, we simulate the BERs of a Rake receiver with conventional MRC and IMOC for different channel conditions. We compare these BERs with the BER that has a diversity gain of an order equal to the number of multipaths (this can be obtained by perfect MPI cancellation) and clearly this BER is the lower bound for the BERs of MRC and IMOC. In our simulations, we increase the number of multipaths to see how the three BERs change. For the same number of multipaths, we choose different channel MIP to see how the three BERs vary.

IV.1 Simulation setups

In our simulation, there are four users (or transportation channels) with the total cumulative power unity. The power of each user is defined as Ec/Ior [13] where Ec is the chip energy and Ior is the total power spectrum density and the modulation scheme is BPSK. The spreading factor (SF), the Doppler frequency, the signal to AWGN noise, and the MIP are listed on the top of the simulation graphs. The simulation results are BER curves of MRC, IMOC and a lower bound as a function of Ec/Ior under different parameters listed on the top of each curve.

IV.2 Simulation results

In Fig. 1, there are two multipath components with 0dB and –10dB average power. We can see IMOC has about 1 dB gain over MRC. The lower bound, due to the diversity gain, has an 8 dB gain over IMOC. In Fig. 2, there are also two multipaths, but the MIP is constant instead of being decayed as in Fig. 1. We can see that the BERs of both MRC and IMOC are higher than those of MRC and IMOC in Fig. 1, while the BER of the bound is lower that that of the bound in Fig.1. The reason for this is that the bound has a diversity gain of an order two, and its BER reaches the minimum when the channel MIP is constant, while the BERs of MRC and IMOC reach the maximum when the channel MIP is constant. In Fig. 3 and Fig. 4, we compare the BERs among MRC, IMOC and the lower bound for four multipaths and with different channel MIP. We can see that MRC and IMOC perform better under the decayed MIP, while the lower bound performs better under the constant MIP for the same reason as explained for Fig. 1 and Fig. 2. In addition, we can see from Fig. 2 and Fig. 4 that, for the same constant channel MIP, the BERs of the two multipaths for both MRC and IMOC in Fig. 2 are lower than those of four multipaths in Fig. 4, while the BER of the two multipaths for the lower bound is higher than that of the four multipaths. This is again due to the same reason, i.e., the lower bound has a diversity gain of a full order while the MRC and IMOC do not due to MPI. Note also that IMOC, even though it is an optimal combining scheme, does not have a much lower BER than MRC and both BERs are much higher than the lower bound. This is because both IMOC and MRC do not have any diversity gain.

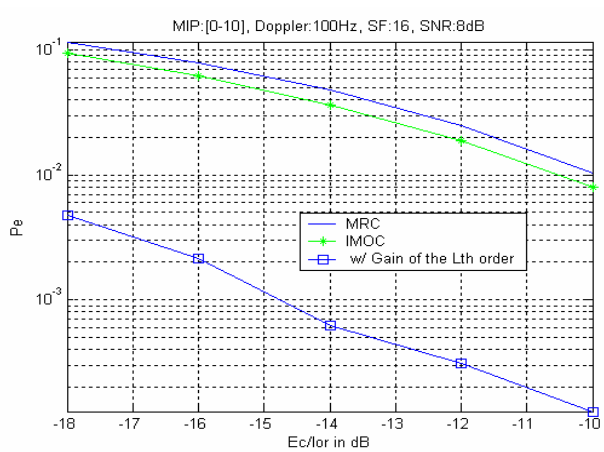

Fig. 1 BER for two decayed (in power) multipaths

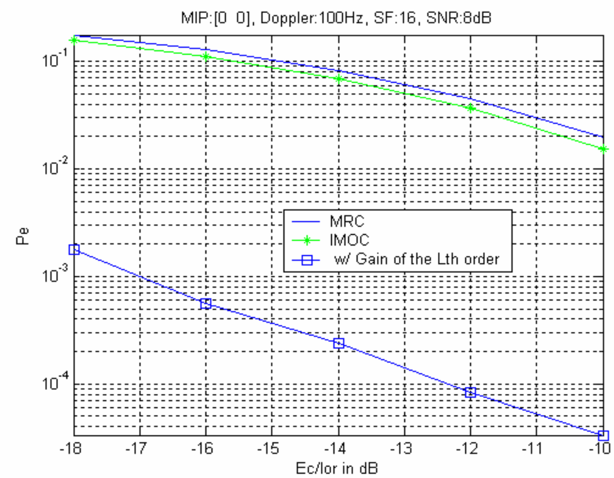

Fig. 2 BER for two multipaths with same power

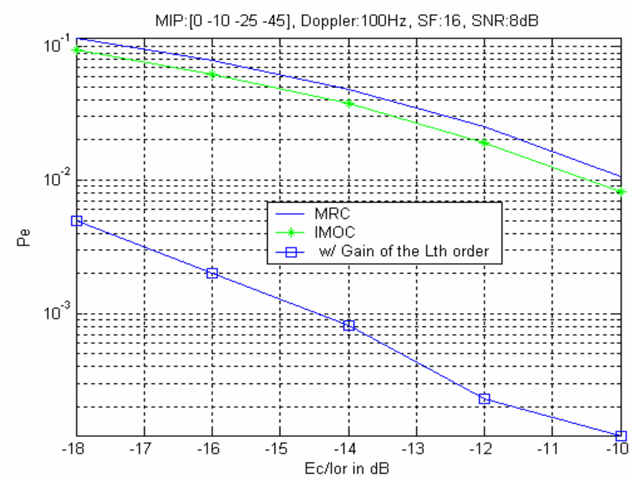

Fig. 3 BER for four decayed (in power) multipaths

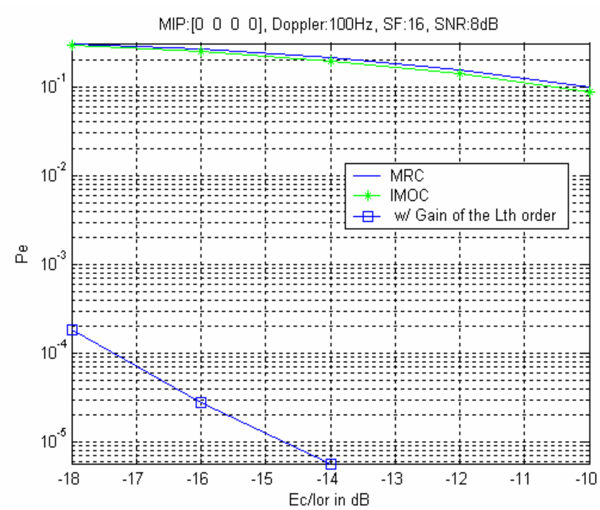

Fig. 4 BER for four multipaths with same power

V. Conclusions

We have proven both in theory and simulations that the performance of an optimum Rake receiver for the CDMA downlink signal does not achieve any diversity gain. On the contrary, its SIR (F distributed) and BER degrade as the number of multipath components increases and the channel MIP changes from delayed to flat. We have also presented the MRC threshold condition for it not to combine the multipaths which will result in both a performance loss and an increase in complexity.

Appendix A: Proof of average BER having its maximum when $c_j = c_i$, for $i \neq j$.

From (12),

$$P_b = \int_0^\infty \cdots \int_0^\infty \frac{1}{\pi}\int_0^{\frac{\pi}{2}} e^{-\frac{1}{(\sin^2\phi)}\sum_{i=1}^{L}\frac{z_i}{\left(\sum_{l\neq i}^{L} z_l\right)}\Gamma_0} d\phi \prod_{j=1}^{L} c_j e^{-(c_1 z_1 + c_2 z_2 + \cdots + c_L z_L)} dz_1 dz_2 \cdots dz_L$$

with the constraint of $1/c_1 + \cdots + 1/c_L = a$. Now we prove that P_b have its maximum when $c_j = c_i$, for $i \neq j$. By Lagrange method, let

$$F(c_1, c_2, \cdots, c_L) = P_b(c_1, c_2, \cdots, c_L) + \lambda(1/c_1 + \cdots 1/c_L - a)$$

if P_b reaches max, then c_i must satisfies:

$$\frac{\partial F}{\partial c_i} = \frac{\partial P_b}{\partial c_i} - \frac{\lambda}{c_i^2} = 0 \quad i = 1, \cdots, L \text{ (A-1)}$$

and it is obvious that $c_i = c_j = a/L$ is a solution to (A-1). Therefore, the extreme of P_b occurs when the MIP is constant. Next we will prove the extreme is a maximum, or the matrix $A = \left(\frac{\partial^2 P_b}{\partial c_i \partial c_j}\right)$ is negative definite, i.e., for any vector $\mathrm{p} \neq 0$ and $\sum_{i=1}^{L} p_i = 0$, $\mathbf{p}^H \mathbf{A} \mathbf{p} < 0$. First let's compute $\frac{\partial^2 P_b}{\partial c_i \partial c_j}$ for $i, j = 1, \cdots, L$:

$$\frac{\partial P_b}{\partial c_i} = \int_0^\infty \cdots \int_0^\infty \frac{1}{\pi}\int_0^{\frac{\pi}{2}} e^{-\sum_{i=1}^{L}\frac{z_i}{(\sin^2\phi)\left(\sum_{l\neq i}^{L} z_l\right)}\Gamma_0} d\phi \left(\prod_{\substack{j=1\\ j\neq i}}^{L} c_j - \prod_{j=1}^{L} c_j z_i\right) e^{-\sum_{i=1}^{L} c_i z_i} dz_1 \cdots dz_L$$

$$= \int_0^\infty \cdots \int_0^\infty \frac{1}{\pi}\int_0^{\frac{\pi}{2}} e^{-\sum_{i=1}^{L}\frac{z_i}{(\sin^2\phi)\left(\sum_{l\neq i}^{L} z_l\right)}\Gamma_0} d\phi \prod_{j=1}^{L} c_j e^{-\sum_{i=1}^{L} c_i z_i}\left(\frac{1}{c_i} - z_i\right) dz_1 \cdots dz_L$$

$$\frac{\partial P_b^2}{\partial c_i^2} = \int_0^\infty \cdots \int_0^\infty \frac{1}{\pi}\int_0^{\frac{\pi}{2}} e^{-\sum_{i=1}^{L}\frac{z_i}{2(\sin^2\phi)\left(\sum_{l\neq i}^{L} z_l\right)}\Gamma_0} d\phi \prod_{j=1}^{L} c_j e^{-\sum_{i=1}^{L} c_i z_i}\left\{\left(\frac{1}{c_i} - z_i\right)^2 - \frac{1}{c_i^2}\right\} dz_1 \cdots dz_L$$

For $i \neq j$,

$$\frac{\partial^2 P_b}{\partial c_i \partial c_j} = \prod_{l=1}^{L} c_l \int_0^\infty \cdots \int_0^\infty \frac{1}{\pi}\int_0^{\frac{\pi}{2}} e^{-\sum_{i=1}^{L}\frac{z_i}{(\sin^2\phi)\left(\sum_{l\neq i}^{L} z_l\right)}\Gamma_0} d\phi \left(\frac{1}{c_i} - z_i\right)\left(\frac{1}{c_j} - z_j\right) e^{-\sum_{i=1}^{L} c_i z_i} dz_1 \cdots dz_L$$

$$p^T A p = \prod_{l=1}^{L} c_l \int_0^\infty \cdots \int_0^\infty \frac{1}{\pi}\int_0^{\frac{\pi}{2}} e^{-\sum_{i=1}^{L}\frac{z_i}{(\sin^2\phi)\left(\sum_{l\neq i}^{L} z_l\right)}\Gamma_0} d\phi \left\{\left[\sum_{i=1}^{L}\left(\frac{1}{c_i} - z_i\right)p_i\right]^2 - \sum_{i=1}^{L}\frac{p_i^2}{c_i^2}\right\} e^{-\sum_{i=1}^{L} c_i z_i} dz_1 \cdots dz_L$$

$$= \left(\frac{L}{a}\right)^L \int_0^\infty \cdots \int_0^\infty \frac{1}{\pi}\int_0^{\frac{\pi}{2}} e^{-\sum_{i=1}^{L}\frac{z_i}{(\sin^2\phi)\left(\sum_{l\neq i}^{L} z_l\right)}\Gamma_0} d\phi \left\{\left[\sum_{i=1}^{L}\left(\frac{L}{a} - z_i\right)p_i\right]^2 - \frac{L^2}{a^2}\sum_{i=1}^{L} p_i^2\right\} e^{-\sum_{i=1}^{L} c_i z_i} dz_1 \cdots dz_L$$

$$= \left(\frac{L}{a}\right)^L \int_0^\infty \cdots \int_0^\infty \frac{1}{\pi}\int_0^{\frac{\pi}{2}} e^{-\sum_{i=1}^{L}\frac{z_i}{(\sin^2\phi)\left(\sum_{l\neq i}^{L} z_l\right)}\Gamma_0} d\phi \left\{\left[\sum_{i=1}^{L}\left(\frac{L}{a} - z_i\right)p_i\right]^2 - \frac{L^2}{a^2}\sum_{i=1}^{L} p_i^2\right\} e^{-\sum_{i=1}^{L} c_i z_i} dz_1 \cdots dz_L$$

$$= \left(\frac{L}{a}\right)^L \int_0^\infty \cdots \int_0^\infty \frac{1}{\pi}\int_0^{\frac{\pi}{2}} e^{-\sum_{i=1}^{L}\frac{z_i}{(\sin^2\phi)\left(\sum_{l\neq i}^{L} z_l\right)}\Gamma_0} d\phi \left\{\left[\sum_{i=1}^{L}\left(\frac{L}{a} p_i - z_i\, p_i\right)\right]^2 - \frac{L^2}{a^2}\sum_{i=1}^{L} p_i^2\right\} e^{-\sum_{i=1}^{L} c_i z_i} dz_1 \cdots dz_L$$

$$= \left(\frac{L}{a}\right)^L \int_0^\infty \cdots \int_0^\infty \frac{1}{\pi}\int_0^{\frac{\pi}{2}} e^{-\sum_{i=1}^{L}\frac{z_i}{(\sin^2\phi)\left(\sum_{l\neq i}^{L} z_l\right)}\Gamma_0} d\phi \left[\left(\sum_{i=1}^{L} z_i\, p_i\right)^2 - \frac{L^2}{a^2}\sum_{i=1}^{L} p_i^2\right] e^{-\sum_{i=1}^{L} c_i z_i} dz_1 \cdots dz_L$$

$$\leq \left(\frac{L}{a}\right)^L \int_0^\infty \cdots \int_0^\infty e^{-\sum_{i=1}^{L}\frac{z_i}{\left(\sum_{l\neq i}^{L} z_l\right)}\Gamma_0} \left[\left(\sum_{i=1}^{L} z_i\, p_i\right)^2 - \frac{L^2}{a^2}\left(\sum_{i=1}^{L} p_i^2\right)\right] e^{-\sum_{i=1}^{L} c_i z_i} dz_1 \cdots dz_L \quad \text{(A-2)}$$

It is hard to prove analytically that (A-2) is less than zero. However, if using the tool of Mathematica one will always find that (A-2) is less than zero.

References:

1. N Kong, I Riphagen, M Lotter and P. van Rooyen, "A Simple and New Optimum Combining Scheme for CDMA Down Link Signals," Accepted for publication on *IEEE Trans on Wireless Commutation* as a letter.
2. S. L. Miller, M. L. Honigand L. B. Milstein, " Performance analysis of MMSE receivers for DS-CDMA in frequency-selective fading," *IEEE Trans Comm*., pp. 1919-1929, Nov. 2000
3. A. Kansal, S. Batalama, and D. Pados, "Adaptive maximum SINR RAKE filtering for DS/CDMA multipath fading channels," *Journal on Selected Areas in Comm.*, Dec. 1998, pp. 1765-1774.
4. J. H. Winters, "OC in digital mobile radio with cochannel interference," *IEEE J. Select Areas Comm*., Vol. SAC-2, no. 4, July 1984.
5. A. Klein, "Data detection algorithms specially designed for the downlink of CDMA mobile radio systems," *Proc. of IEEE VTC'1997*
6. J. Yang and G. Li, "A decision-feedback equalizer with tentative chip feedback for the downlink of WCDMA", *Proc. of IEEE ICC'2002*
7. P. Komulainen, M. J. Heikklila and J. Lilleberg , "Adaptive channel equalization and interference suppression for CDMA downlink," IEEE 6th Int. Symp. on Spread-Spectrum Tech. & Appl., NJIT, New Jersey, Sept. 6-8, 2000
8. G. E. Bottomley, "Optimizing the RAKE receiver for the CDMA downlink," *Proc. IEEE VTC*'93, pp. 742-745
9. J. Proakis, *Digital Communication.* New York: McGraw-Hill, 1983
10. G. Kutz and A. Chass, "On the performance of a practical downlink CDMA generalized RAKE receiver," Proceedings of *IEEE VTC* fall 2002
11. M.-S. Alouini and M. K. Simon, "An MGF-based performance analysis of generalied selection combining over Rayleigh fading channels," *IEEE Trans. Commun*., vol. 48, pp. 401-415, Mar. 2000
12. N. Kong and L. B. Milstein, "SNR of generalized diversity selection combining with nonidentical Rayleigh fading statistics", *IEEE Tran. On Comm*. Aug. 2000, pp 1266-1271
13. J. W. Craig, "A new, simple, and exact result for calculating the probability of error for two-dimensional signal constellations," *in Proc. IEEE MILCOM'91*, pp.571-575
14. 3GPPTS34.121, "Terminal Conformance Specification; Radio Transmission and Reception (FDD)"