PARTITIONED MARKOV MODELS FOR DS/SSMA

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I. Introduction

During the last five years, there has been increasing interest in DS/SSMA systems to provide the higher system capacity required by both mobile and wireless access networks. One of the most important system parameters in such an access network, is the average BER as a function of the number of users, K, and the processing gain, N. Several papers have been written on ways to determine the BER. These range from mathematical bounds [1] on the average BER to approximations of the average BER [2, 3, 4]. Although these methods provide accurate measures of the BER, it does not provide any information on the way in which the errors occurred (ie. whether the channel exhibits burst-like errors, or if the channel is a pure memoryless channel).

It has been shown that the use of Forward Error Correction (FEC) strategies dramatically increase the capacity of a DS/SSMA system [4, 5]. In order to determine the optimal FEC strategy to employ, it is important to know the manner in which errors occur on the DS/SSMA channel. This information can only be gathered by performing exhaustive simulations which accurately model a true DS/SSMA system. Because of the complex nature of a DS/SSMA system, these simulations are very time consuming. In the simulation of an asynchronous DS/SSMA system, the random delays between subscribers must constantly be updated, and, if fading is present, the transmission of each individual user must be weighed by an independent fading process. Also, the simulation time is very dependent on the number of users and the processing gain.

In this paper, we shall present an alternative way of simulating a DS/SSMA system using the Fritchman channel model [6]. The advantage of this method is that, once a model of the channel has been created from simulated error statistics, the DS/SSMA system can be simulated very quickly, with the simulation time being independent of the number of users and the processing gain. Thus, a typical application for this simulation technique would be to evaluate different FEC coding strategies over a specified channel.

In section 2, the general multiple access model is described, while section 3 introduces the Fritchman model. It is shown how this model can be applied to the simulation problem described in section 2. Section 4 presents simulation results obtained using exhaustive simulation as well as the Fritchman technique. In section 5, it is shown how the Fritchman model can be used in the design of error correction strategies. Finally, some conclusions are drawn in section 6.

II. The General Multiple Access Model

The multiple access system shown in Figure 1 will be used as a basis for the simulations performed in this paper [7]. The theoretical model assumes the presence of K users, where the desired transmitter and receiver pair ($k = 1$) is randomly delayed relative to all other channels. The data signal of each user is denoted by $b_k(t)$, and the spreading sequence by $a_k(t)$, with delays, denoted by $\tau_k(t)$ uniformly distributed over one bit period $(0, T)$. The values of the random delays, τ_k, are changed every T_f seconds, to simulate burst-mode transmissions where the relative delays (and consequently the relative phase relationships) between the different spreading sequences $(a_k(t))$ are constantly changing. The transmitter-receiver pair is assumed to be perfectly synchronised, and the modulation method used is assumed to be Differentially Encoded Coherent Phase Shift Keying (DE-CPSK). Transmissions from other users have random phase offsets, θ_k, relative to the $k = 1$ pair, with θ_k uniformly distributed over the interval $(0, 2\pi)$. Additive White Gaussian Noise (AWGN), $n(t)$, with two-sided power spectral density of $\eta/2$ W/Hz is added to the channel. The processing gain of the system is assumed to be N, where $N = T/T_c$, and T_c denotes the duration of a single chip.

III. The Fritchman Model

The Fritchman channel [6] has been applied to a wide variety of problems in digital communications.

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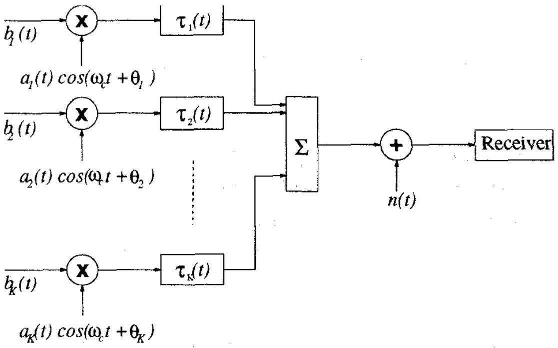

Fig. 1. General structure of the DS/SSMA system

Tsai [8] applied the model to H.F. channels, Knowles and Drukarev [9] and Drukarev and Yiu [10] to the modeling of magnetic recording channels and Oosthuizen et. al. [11] to the modeling of Hamming code error detection, misdetection and decoding error events. In this paper, the Fritchman model will be applied to the modeling of MUI in DS/SSMA systems.

Figure 2 shows the state partitioning of the generalized Fritchman channel model.

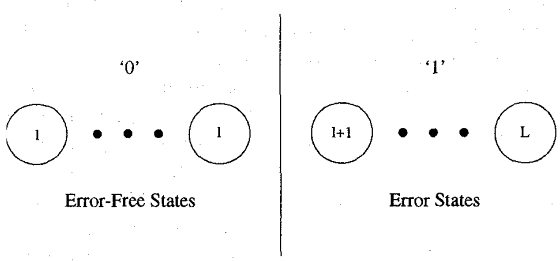

Fig. 2. State partitioning of the generalized Fritchman model

Most of the applications using this model use only one error state. This simplifies the model considerably and causes its statistical process to become a renewal process. The parameters of this model are uniquely determined by the Error-Free Run (EFR) distribution or $P(0^m|1)$, the probability that an error bit (denoted by 1) will be followed by at least m successive error-free bits (denoted by 0^m, where 0 denotes an error-free bit).

The model parameters (transition probabilities) are determined from $P(0^m|1)$ by fitting an exponential curve of the form

$$y(x) = A_1 e^{-\alpha_1 x} + A_2 e^{-\alpha_2 x} + \ldots + A_{L-1} e^{-\alpha_{L-1} x} \quad (1)$$

to $P(0^m|1)$, where x denotes the error-free run length. This model can be represented by the following state transition matrix:

$$P = \begin{bmatrix} p_{11} & 0 & 0 & \cdots & p_{1L} \\ 0 & p_{22} & 0 & \cdots & p_{2L} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ p_{L1} & p_{L2} & \cdots & p_{LL-1} & p_{LL} \end{bmatrix} \quad (2)$$

The transition probabilities in (2) are derived from (1) using the method described in [8].

The Fritchman model presented takes into account the effects of the Differential Encoding (DE) and Differential Decoding (DD) processes, the MUI as well as AWGN (see Figure 3).

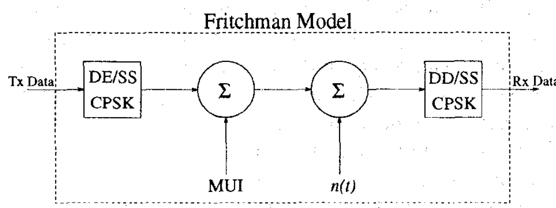

Fig. 3. General Fritchman channel model

Using the Fritchman model to simulate a DS/SSMA system, requires the calculation of the model parameters using the aforementioned technique, and data generated by exhaustive simulation. The advantage of the Fritchman model based simulation, is that the exhaustive simulation needs to be carried out only once. Thereafter, the Fritchman model parameters are known, and can be used for subsequent simulations and determination of FEC coding performance parameters.

IV. Simulation Results

In this section, it is shown how the models described in section III. can be used to simulate a DS/SSMA system. The process of determining the model is given by he following steps:

1. Generate EFR statistics by exhaustive simulation or measurement from a physical system.
2. Determine model parameters using the method described in [8].
3. Simulate system using Fritchman model.

The Fritchman model can then be used to determine a number of error statistics [13]. In this paper, we will concentrate on the *error-free run distribution* and the *burst length distribution*. The EFR distribution was defined in section III.. The burst length distribution refers to the probability of receiving burst errors with burst lengths of at least B bits. Errors are said to have occurred in a burst when the local error probability exceeds a specified threshold, in this case 0.1. Whereas the EFR distribution is used to determine the parameters of the Fritchman model, the burst length distribution is used to help determine the optimum error control strategy for a specific system.

The DS/SSMA system was simulated using exhaustive simulation and the Fritchman model under the following conditions:

$K = 10,25$
$N = 63$
$E_b/N_o = 15$ dB

In the case of $K = 10$, a Fritchman model with three error-free states was used, and, in the case of $K = 25$ users, a model with two error-free states was employ ed. Both Fritchman models have only one error state. The corresponding EFR and b urst length distributions are shown in Figures 4, 5 and 6 respectively. The EFR distributions shown in Figure 4 compares the EFR distribution of the simulated and modelled DS/SSMA systems to that of a Binary Symmetrical Channel (BSC) operating at a comparable error rate. If the Gaussian assumption for the MUI [7] on the DS/SSMA channel was valid, then the EFR distribution for the DS/SSMA system and that of the BSC should have coincided. The BER determined from both the exhaustive simulation and the Fritchman models, was found to be equal to 3.8×10^{-4} when $K = 10$ and 1.3×10^{-2} when $K = 25$. Using these values as parameters for the BSC, the results shown in Figure 4 clearly indicates that the error characteristics of the DS/SSMA channel is non-Gaussian. In fact, when the number of users, K, is small (i.e. 10), the simulated and modelled EFR distributions differ substantially from that of the BSC, and looks like a channel with memory. As the number of users increase, the Gaussian assumption becomes more valid via the Central Limit Theorem [14].

Although the Fritchman model accurately predicts the BER and the EFR distributio n, the distribution of burst lengths is modelled less accurately. Because data in the DS/SSMA system is differentially encoded, most of the decoded errors occur in pairs. This is clearly shown by the high relative frequency in the occurrence of paired errors generated by the exhaustive simulation (see Figures 5 and 6 at burst length values equal to 2). In both cases, the Fritchm an model generated substantially less paired errors than the exhaustive simulation. Using a Fritchman model with two error states, can partly address this problem. However, models using two error states normally force paired errors, and cannot simulate single errors. Also, these models normally use only two error-free states to model the error process [12]. Because of the non-Gaussian nature of the EFR distribution, it is necessary to use at least three error-free states to

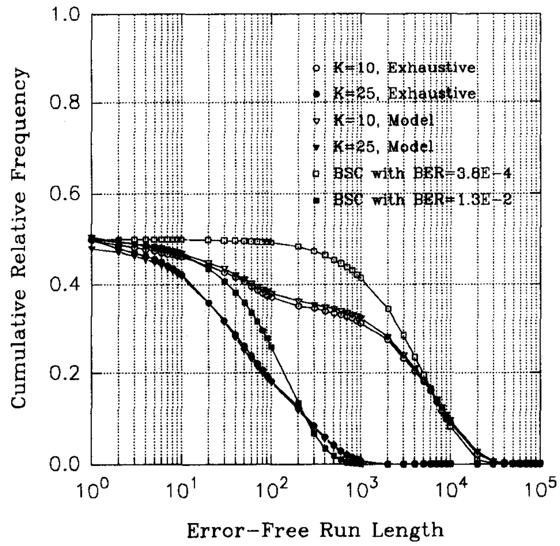

Fig. 4. Error-Free Run Distributions for $K = 10$ and $K = 25$ with $E_b/N_o = 15dB$ and $N = 63$

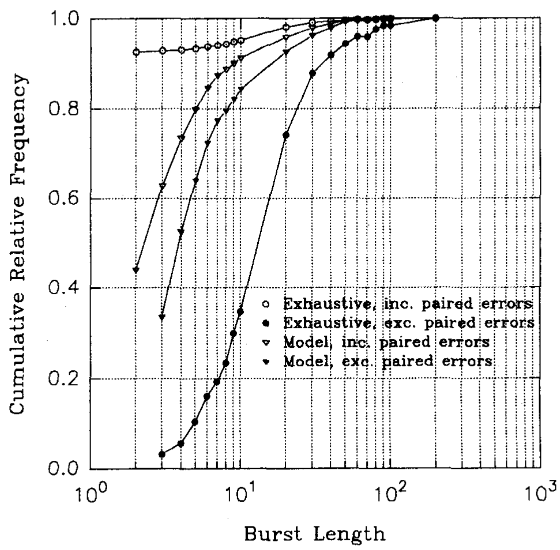

Fig. 5. Burst Length Distribution for $K = 10$ with $E_b/N_o = 15$ dB and $N = 63$

accurately model the EFR distribution (as was done in this case). Furthermore, if paired errors are excluded from the burst length distribution calculation (this as a valid assumption, as we know that a lot of paired errors will occur), the Fritchman model becomes a pessimistic model of the burst length distribution. In this case, the model predicts more bursts of short lengths than the exhaustive simulation gen-

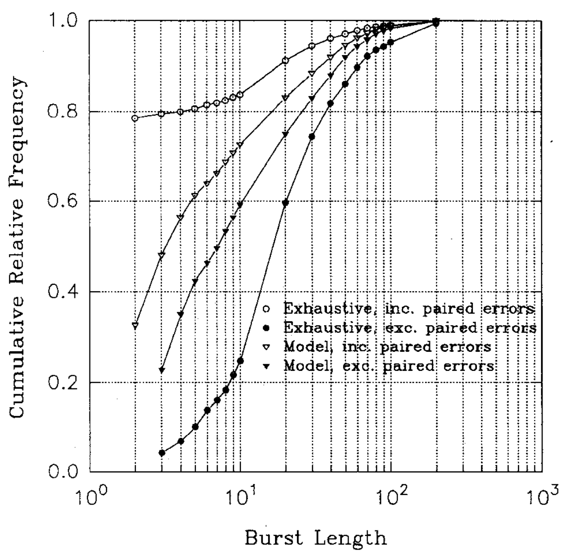

Fig. 6. Burst Length Distribution for $K = 25$ with $E_b/N_o = 15dB$ and $N = 63$

erates. In both cases (including and excluding paired errors), however, the maximum lengths of bursts are modelled accurately.

V. Applications to Error Control Design

The use of Error Control Coding (ECC) is invaluable in a DS/SSMA system. Not only does it ensure the integrity of the data link, but it also increases the system capacity [4, 5]. In order to determine optimal error control strategies, it is important to have available values for the block error probability, $P(m,n)$ where m denotes the number of errors in a block of length n. The values for $P(m,n)$ can easily be determined from the Fritchman model using the method described in [15], namely

$$P(m,n) = \sum_{i=1}^{L} P_i f_i(m,n) \tag{3}$$

where P_i is the probability of being in state i, also termed the stationary state probability of state i. The distribution $f_i(m,n)$ is given by the following recurrent relationship:

$$f_i(m,n) = \sum_{j=1}^{k} p_{ij} f_i(m,n-1) + \sum_{j=k+1}^{L} p_{ij} f_i(m-1,n-1) \tag{4}$$

where $i = 1, 2, \ldots, L$ and,

$$f_i(m,n) = 0 \quad \text{for} \quad m > n$$
$$f_i(m,n) = 0 \quad \text{for} \quad n < 0 \text{ or } m < 0$$
$$f_i(0,0) = 1$$

Using these values for $P(m,n)$, the probability of having m or more errors in a block of length n, denoted by $P(\geq m,n)$, can easily be calculated. These values can then be used in the selection of appropriate error correcting codes [16]. Values for $P(\geq m,n)$ calculated using the Fritchman model for different values of K and n are shown in Figures 7 and 8. From these figures, it is clear that an error correction strategy suitable for a system with a low number of users will not be adequate for a system with a high number of users. Also, the figures give an indication of appropriate block lengths to consider when using block codes for error correction.

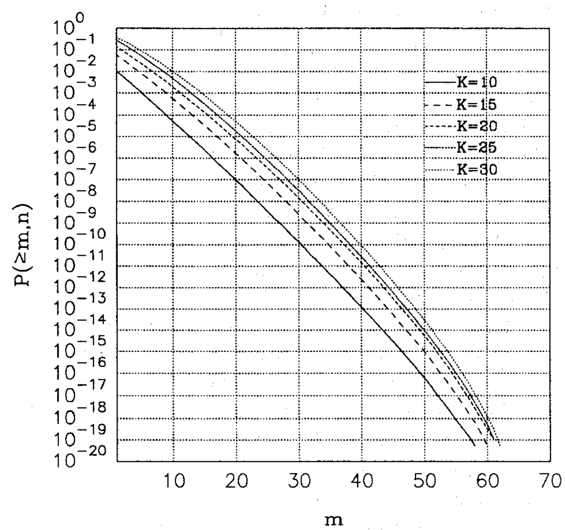

Fig. 7. $P(\geq m,63)$ for a DS/SSMA system with different values of K

VI. Conclusions

In this paper we have shown that the Fritchman channel model can be directly applie d to the simulation of a DS/SSMA system. This simulation method has the advantage t hat, once the model parameters are known, simulations can be performed in a very short time compared to that of an exhaustive simulation. Whereas the simulation time of an exhaustive simulation increases dramatically for large values of N and K the simulation time for the Fritchman model is independent of both N, K and E_b/N_o. The simulation time of the Fritchman model is also substantially less, as the Fritchman model is a pure state machine. This means that only a single random value must be calculated and compared to some decision threshold in the simulation of a data bit. The results

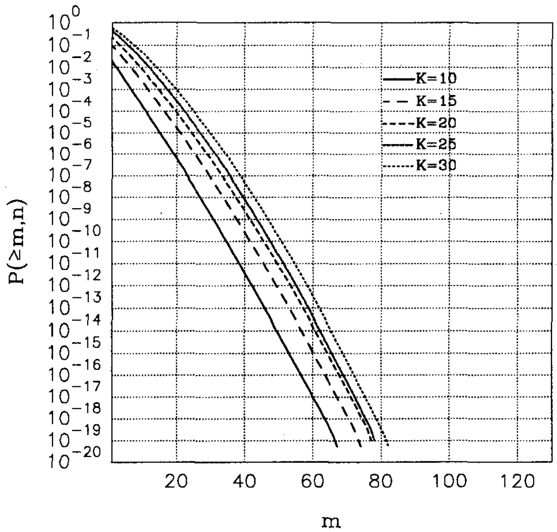

Fig. 8. $P(\geq m, 127)$ for a DS/SSMA system with different values of K

obtained examining the EFR distributions, also point to the non-validity of the assumption that the MUI can be modelled as a Gaussian process. The EFR clearly shows that ECC suitable for AWGN channels will not be optimal for a DS/SSMA system. In fact, because the EFR distribution looks similar to that of a channel with memory, ECC strategies suitable for channels with memory might be more appropriate, depending on the number of users operating in the system.

References

[1] M.B. Pursley, D.V. Sarwate and W. Stark, "Error Probability for Direct-Sequence Spread-Spectrum Multiple-Access Communications - Part I: Upper and Lower Bounds", *IEEE Trans. Communications*, vol. COM-30, pp. 975-984, May 1982.

[2] E.A. Geraniotis and M.B. Pursley, "Error Probability for Direct-Sequence Spread-Spectrum Multiple-Access Communications - Part II: Approximations", *IEEE Trans. Communications*, vol. COM-30, pp. 985-95, May 1982.

[3] D. Laforgia, A. Luvison and V. Zingarelli, "Bit Error Rate Evaluation for Spread-Spectrum Multiple-Access Systems", *IEEE Trans. Communications*, vol. COM-32, pp. 660-669, 1984.

[4] P.G.W. van Rooyen, *Maximum Entropy Analysis of a DS/SSMA Diversity System*, D.Eng Dissertation, Rand Afrikaans University, Johannesburg, 1994.

[5] A.J. Viterbi, "Very Low Rate Convolutional Codes for Maximum Theoretical Performance of Spread-Spectrum Multiple-Access Channels", *IEEE J. Selected Areas in Communications*, vol. 8, pp. 641-649, May 1990.

[6] D.B. Fritchman, "A Binary Channel Characterization using Partitioned Markov Chains", *IEEE Trans. on Information Theory*, vol. IT-13, pp. 221-227, 1967.

[7] M.B. Pursley, "Performance Evaluation for Phase-Coded Spread-Spectrum Multiple-Access Communications - Part I: System Analysis", *IEEE Trans. Communications*, vol. COM-25, pp.795-799, August 1977.

[8] S. Tsai, "Markov characterization of the H.F. channel", *IEEE Trans. on Communication Technology*, vol. COM-17, pp. 24-32, Feb. 1969.

[9] M.D. Knowles and A.I. Drukarev, "Bit Error Rate Estimation for Channels with Memory", *IEEE Trans. on Communications*, vol. 36, pp. 767-769, June 1988.

[10] A.I. Drukarev and K.P. Yiu, "Performance of Error-Correcting Codes on Channels with Memory", *IEEE Trans. on Communications*, vol COM-34, pp.513-521, June 1986.

[11] D.R. Oosthuizen, H.C. Ferreira and F. Swarts, "On Renewal Inner Channels and Block Code Error Control Outer Channels," in *Proc. IEEE GLOBECOM'89*, pp. 1703-1707, November 27-30, 1989.

[12] Y. Zhang, *Markov Channel Models for Magneto-Optical Disks*, Ph.D. Dissertation, Worcester Polytechnic Institute, Worcester MA., 1990.

[13] F. Swarts, *Markov Characterization of Fading Channels*, M.Eng Thesis, Rand Afrikaans University, Johannesburg, 1991.

[14] A. Papoulis, *Probability, Random Variables, and Stochastic Processes*, Singapore: McGraw-Hill, 1991.

[15] L.N. Kanal and A.R.K. Sastry, "Models for Channels with Memory and Their Applications to Error Control", in *Proc. IEEE*, vol. 66, pp.724-744, July 1978.

[16] G.C. Clark, Jr. and J. Bibb Cain, *Error-Correction Coding for Digital Communications*, New York: Plenum Press, 1981.