# Modeling Spatial Aspects of Cellular CDMA/SDMA Systems

Michiel P. Lötter and Pieter van Rooyen

***Abstract*— In this letter, probability density functions (pdf's) describing various physical aspects of hybrid cellular code-division multiple-access/space-division multiple-access (CDMA/SDMA) networks are derived. All mobile users are considered to be surrounded by a Gaussian bell shape of scattering elements, and a general rule of thumb for the standard deviation of the scattering elements is proposed. Second, in order to describe the distribution of mobile users in the cell, a general pdf describing mobile user distributions is proposed. Using the mentioned distribution a general pdf for the angle-of-arrival (AOA) of signals at the CDMA base station is derived.**

***Index Terms*—Antenna array, CDMA, channel model, SDMA.**

## I. Introduction

SDMA systems rely on the use of adaptive narrow-beam antennas and the nonhomogeneous distribution of users in a cellular system to increase system capacity. In particular, adaptive antenna arrays have been receiving a lot of attention in literature [1], [2]. As SDMA systems rely on the spatial information of users in a cell to increase system performance, assumptions made in this regard are extremely important.

One of the most important spatial assumptions relates to the pdf of the angle-of-arrival (AOA) of multipath signals at a receiver. This distribution impacts on the antenna design as well as system performance and needs to be modeled accurately. As all users in a code-division multiple-access (CDMA) system are transmitting to the base station at the same time, the main factors influencing the pdf of the AOA are the distribution of local scattering elements, as well as the distribution of mobile users throughout the cell. In Section II, the assumption of Gaussian distributed local scattering elements is explained and a new rule of thumb to determine the standard deviation of the scattering elements is proposed. The distribution as mobile users is addressed in Section III where a general pdf capable of describing various mobile user distributions is proposed. In Section IV the above information is combined to derive an expression for the pdf of the AOA at the base station of a hybrid CDMA/SDMA system. Finally, the letter is concluded in Section V.

## II. Local Scattering Elements

It is assumed that each mobile is surrounded by a large number of scattering elements. The scattering elements can be described either as a ring of scatterers around the mobile [1] or as a disc of scatterers around the mobile [3]. However, in [4] it is shown that scattering elements can best be described by a Gaussian bell shape, as shown in Fig. 1. In this case it is assumed that the majority of scattering points are situated close to the mobile, with the density of scattering points decreasing as the distance from the mobile is increased.

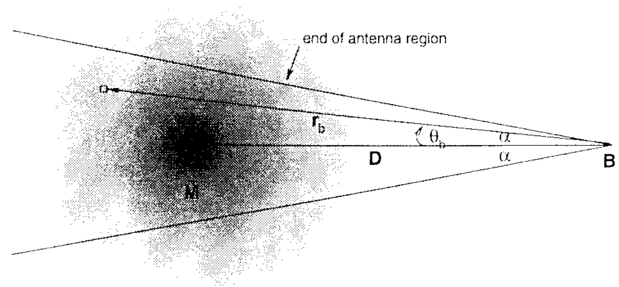


Fig. 1. Modeling of scattering elements using a Gaussian approach.

In Fig. 1, the mobile $M$ is separated from the bases station $B$ by an arbitrary distance, $D$. The base station employs a directional antenna with beam width $2\alpha$ to illuminate the mobile, as well as a portion of the scatterers around the mobile. In this case, the density of the scattering elements around the mobile at a distance $r_b$ and angle $\theta_b$ from the base station, can be described by the bivariate Gaussian distribution [4]

$$P_{\text{scat}}(r_b, \theta_b) = \frac{1}{2\pi\sigma^2} e^{(-r_b^2 + D^2 - 2r_b D \cos\theta_b)/2\sigma^2} \tag{1}$$

where $\theta_b$ is measured from the horizontal in Fig. 1.

Assuming that the beam width of the antenna structure is known, the pdf of the AOA of signals at the base station can be determined. The general assumption used is that transmissions to and from mobiles may reflect from any scattering point in the cell with equal probability, causing an uniform distribution for the AOA. This assumption is only true if the base station antenna beam illuminates all scattering elements in the cell and when the scattering elements are uniformly distributed [3]. Clearly this is not the case for narrow beam antennae as is shown in Fig. 1. In order to calculate the pdf of the AOA's, the cumulative distribution of scattering elements as a function of $\theta_b$ is required. This cumulative distribution, or the addition of more scattering elements as a function of angle is then

$$W_{\theta_b} = \int_{-\alpha}^{\alpha} \int_{0}^{\infty} \frac{1}{2\pi\sigma^2} e^{(-r_b^2 + D^2 - 2r_b D \cos\theta_b)/2\sigma^2} \, dr_b \, d\theta_b. \tag{2}$$

The probability of receiving a signal with a certain AOA is then directly related to the density of scatterers in the specific

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The authors are with AlcaTel AlTech Telecoms, Boksburg, South Africa.

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direction, or the derivative of (2) with respect to $\theta_b$. Therefore

$$p_{\Theta_b}(\theta_b) = \frac{d}{d\theta_b} W_{\theta_b} = \int_0^{\infty} \frac{A}{2\pi\sigma^2} e^{(-r_b^2 + D^2 - 2r_b D \cos\theta_b)/2\sigma^2}\, dr_b \quad (3)$$

where $A$ is a normalizing constant such that $\int_0^{2\pi} p_{\Theta_b}(\theta_b)\, d\theta_b = 1$. By algebraic manipulation, (3) can be simplified to

$$p_{\Theta_b}(\theta_b) = \frac{A}{2\sqrt{2\pi}\sigma} e^{D^2(\cos^2\theta_b - 1)/2\sigma^2} \operatorname{erfc}\left(\frac{-D\cos\theta_b}{\sqrt{2}\sigma}\right), \quad -\alpha \le \theta_b \le \alpha \quad (4)$$

where $\operatorname{erfc}(x)$ is the well known complementary error function. Whereas (4) represents the post-antenna pdf of the AOA, it should be noted that the pre-antenna pdf of the AOA seen at the base station can be seen as a special case of (4) when $\alpha = 180°$.

In addition to the beam width of the antenna array, the standard deviation of the local scattering elements needs to be known in order to use (4) in calculations. A general rule of thumb for the first order approximation of the standard deviation is proposed by the authors. As a basis for the derivation of the rule, consider the multipath intensity profiles recommended in COST-207 [6]. All things equal, it can be assumed that the stronger multipath echoes that arrive at the base station shortly after the line-of-sight (LOS) signal are due to scattering points close to the mobile (shorter transmission path length hence lower path loss factors), and that multipath echoes with larger delay values are due to scattering points further from the mobile. For the case of a typical urban area, the intensity of multipath elements have dropped by halve after 0.69 $\mu$s [6]. This delay value translates to a path length difference of 207 m. Typical microcells have cell radii in the order of 1 km, yielding a typical ratio of path length difference to cell radius of approximately $0.2R$, where $R$ is the radius of the cell in meters. The authors propose to use this value as the standard deviation for the Gaussian distribution of the local scattering elements under LOS microcellular conditions.

Shifting the focus to non-LOS microcellular environments, the mobile signal relies only on reflected signal to reach the base station and consequently the standard deviation of the scattering elements appears to be larger. However, in macrocellular scenarios each mobile user sees a substantially smaller proportion of the scattering elements in the cell as the base station and the mobile is separated by much larger distances. Based on empirical evidence and standard channel models, a rule of thumb for the standard deviation of the Gaussian bell shape described in (1) can then be formulated as $\sigma = 0.34R$ for non-LOS microcellular conditions; $\sigma = 0.2R$ for LOS microcellular conditions and $\sigma = 0.1R$ for macrocellular conditions.

The pdf of the AOA of a single user system for typical macro, LOS micro, and non-LOS microcells are shown in Fig. 2. In the figure, $D = 10\ 000$ m and $D = 1000$ m, respectively, for the macro and different microcellular scenarios, with the antenna beam width ($2\alpha$) equal to 10° and local scattering element standard deviation, determined by the rule of thumb, equal to $\sigma = 1000$ m, $\sigma = 200$ m, and $\sigma = 340$ m. From the figure it is clear that the assumption of uniform AOA is not valid.

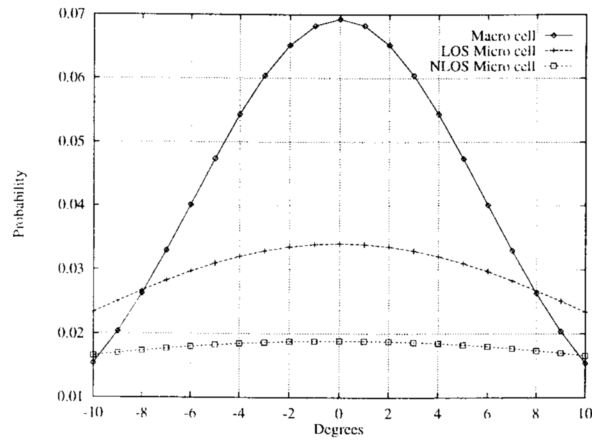


Fig. 2. PDF of the AOA for macro, micro, and pico cellular scenarios.

If site specific propagation data is available, it is possible to calculate more exact values for the local scatterer variance. In [4] it is shown that the crosscorrelation of the fading between antenna elements spaced by $d$ meters is

$$p(d) = e^{-(4\pi d^2 \sigma^2_{\text{angular}}/\lambda^2)} \quad (5)$$

where $\lambda$ denotes the wavelength of the carrier frequency and $\sigma^2_{\text{angular}}$ denotes the variance of the angular spread of signals arriving at the base station antenna. From (5) and the well known formula for the length of an arc, the normalized (with respect to distance from the base station) standard deviation of the scattering elements can be calculated as

$$\sigma = \sqrt{\frac{\lambda^2 \ln(p(d))}{-4\pi d^2}}. \quad (6)$$

From (6) and correlation results presented in [8], the local scatterer variance for the case of a rural macro cell can be calculated as $0.113R$, and for the case of a non-LOS microcell as $0.337R$.

## III. Mobile Location Distribution

In addition to the modeling of the AOA due to scattering elements, the position of mobiles in the CDMA cellular system have a significant influence on the pdf of the AOA of signals at the base station. The position of a mobile user in the cellular structure is fully defined by its distance from the reference base station $r$, and its angle, $\phi_o$, measured from some reference, both of which can be considered random variables (see Fig. 3). True pdf's for both the angular and distance distributions of mobiles vary depending on the situation and various approximations are used [9]. In general, these assumptions are either not a good representation of real world scenarios or are somewhat inflexible in describing different situations. Specifically, a single pdf applicable to many scenarios would be extremely useful. Thus, in terms of the angular distribution

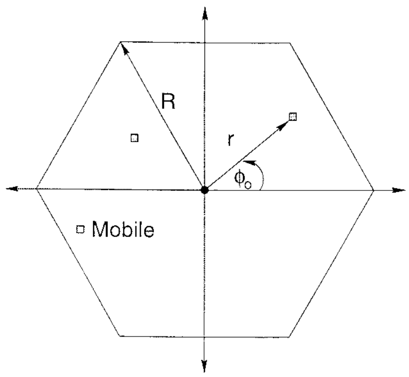


Fig. 3. Modeling the location of mobiles in a cellular system.

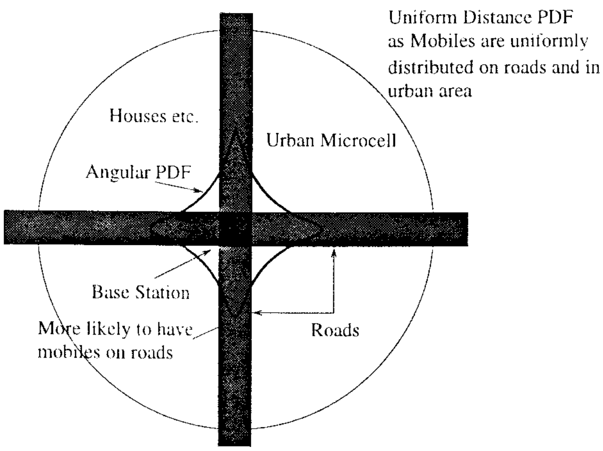


Fig. 4. Qualitative description of the PDF of the mobile distributions in a typical urban microcellular scenario.

of mobiles the authors propose a pdf of the form

$$p_{\Theta_o}(\theta_o) = \frac{1}{A_{\text{norm}}}\left[1 + \sum_{l=1}^{N_{\text{peak}}} \gamma_l \left[\text{rect}\left(\frac{w_l\theta_o}{\pi} - \alpha_l\right) + \text{rect}\left(\frac{w_l\theta_o}{\pi} - \alpha_l - 2\pi\right)\right] \cdot \cos^2(w_l\theta_o - \alpha_l)\right], \qquad 0 \le \theta_o \le 2\pi \tag{7}$$

where $A_{\text{norm}}$ is a normalizing factor to ensure that $\int_0^{2\pi} p_{\Theta_o}(\theta_o)\, d\theta_o = 1$ and $N_{\text{peak}}$ is the number of peaks in the pdf. The factor ($N_{\text{peak}}$) is a measure of the angular clustering of mobiles in a cell. Clearly, if $N_{\text{peak}} = 0$, (7) denotes a uniform angular subscriber distribution. Furthermore, $\text{rect}(x) = 1$ if $|x| < 1/2$ and zero elsewhere, with $w_l$ an integer controlling the width of peak $l$. Typically, values for $w_l$ will be chosen to yield angular peaks of different maximum and minimum widths. The angular location of peak $l$ is given by $\alpha_l$, whilst the relative size of each peak is determined by $\gamma_l$.

As an example, consider Fig. 4, which depicts a typical urban scenario where two main roads intersect with a base station situated at the crossing. As these roads carry a much larger portion of the total mobile traffic than other areas within the coverage region of the cell, the probability of receiving transmission with angles of incidence of 0. $\pi/2$, $\pi$, and $3\pi/2$ rad is substantially higher, as the concentration of mobiles in these specific directions are substantially higher. The pdf of the location of mobiles for this scenario is shown in Fig. 5, where the angular width of a street is taken to be 10°.

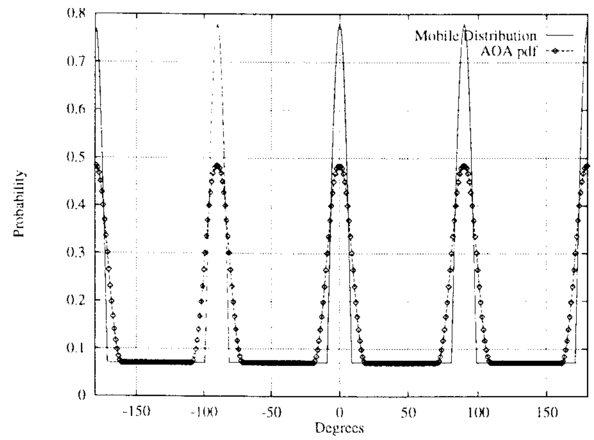


Fig. 5. PDF of mobile distribution, as well as the AOA distribution, at the base station for the conditions shown in Fig. 2 (microcell with $\sigma = 0.2R$, $R = 1000$ m).

## IV. PDF of the Angle of Arrival

If no scattering around the mobile is assumed (the pdf presented in (3) is a delta function), then the pdf of the AOA at the base station would be exactly equal to the pdf of the mobile locations presented above. This is clearly not the case, and hence a pdf describing the AOA of the signals at the base station must take both local scattering and the location of mobiles into account. Then, the pdf of the AOA can be calculated by noting that

$$p_{\Theta}(\theta) = p_{\Theta_b}(\theta_b) * p_{\Theta_o}(\theta_o) \tag{9}$$

where $*$ denotes convolution.

Fig. 5 depicts the convolution of the pdf of the AOA for a typical microcellular case with local scattering effects as depicted in Fig. 2 and the subscriber distribution depicted in Figs. 4 and 5. Clearly it can be seen that the pdf is not uniform as is generally assumed.

## V. Conclusions

In this letter the pdf of the AOA of signals at a base station in a cellular CDMA/SDMA system was calculated. This pdf is required to calculate various parameters of a cellular access system, such as BER, outage probability and Doppler spectrum. A new expression for the distribution of users in a cellular system was presented and the expression was used to show the influence of the user distribution on the pdf of the AOA in hybrid CDMA/SDMA systems. Furthermore, assuming that all users are surrounded by Gaussian distributed local scattering elements, a rule of thumb for the standard deviation of scattering elements in micro- and macrocellular systems was derived.

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