MATHEMATICAL CHANNEL MODELS FOR SPREAD SPECTRUM DATA COMMUNICATIONS

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ABSTRACT

The results of an experimental investigation undertaken for *Direct Sequence Spread Spectrum* under controlled additive white Gaussian noise, multi-user and multi-path conditions, employing coherent DQPSK as modulation scheme, is presented. A discrete partitioned Markov channel model, as proposed by Fritchman, is parameterized to generate an error sequence statistically similar to that of spread spectrum under multi-path conditions, with processing gain and signal- to-noise ratio (SNR) as variable parameters to the model. abstract

1. INTRODUCTION

1.1 The Spread Spectrum Concept

Direct Sequence Spread Spectrum (DS/SS) techniques for use over low signal-to-noise ratio channels are well understood and documented [1], [3], [7], [10]. We briefly summarize the salient features.

A binary digital data sequence producing digits at the rate of R_d digits per second is added modulo-two to a predetermined pseudo-noise (PN) sequence of binary digits occurring at a rate of $R_c \gg R_d$ digits per second. The resulting digit stream is then used to phase modulate an RF carrier and this is transmitted over the given channel. At the receiver, and after synchronization, the in-phase modulo-two addition of a replica of the PN-sequence [5] to the received demodulated waveform will remove the PN-sequence, resulting in a data sequence approximating that of the transmitter.

The quality of this approximation, as expressed by the bit error rate in the receiver output, is normally a function of the so called *Processing Gain* (PG), defined by

$$PG = \frac{R_c}{R_d} , \tag{1}$$

and the signal-to-noise ratio in the channel. In fact, in an AWGN channel of symbol energy to noise power spectral density E_s/N_0, a binary PSK system will yield a BER equal to that of an unspread channel with a signal-to-noise ratio given by

$$\frac{E_b}{N_0} = \frac{E_s}{N_0} \times PG \tag{2}$$

Figure 1 illustrates the probability of a binary digit error for conventional coherent DQPSK signalling, as derived in [6], and compared with DQPSK spread spectrum ($PG = 125$), as characterized by (2), under ideal AWGN conditions. From this graph it is conclusive that the processing gain, realized by spread spectrum, improves the BER for a given SNR. Also displayed is measured error probabilities of spread spectrum with $PG = 125$ and $PG = 64$. As can be seen a difference of about $4dB$, at high signal- to-noise, exists between the expected and experimental values. At low signal-to-noise the difference gets as high as $6dB$. This is due to implementation losses such as modulation distortion, tracking jitter etc., and compares favourably with results in [4].

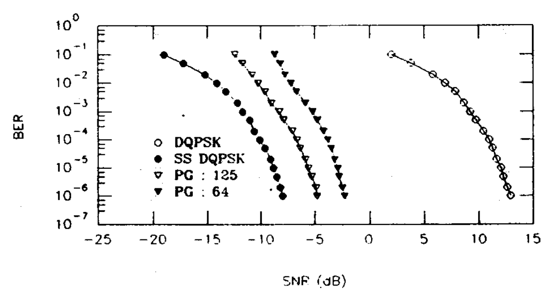

Figure 1: BER Comparison for a DQPSK System

Considering implementation losses when conducting system simulations are thus meaningful. However, models of Spread Spectrum systems, used to simulate (2), often ignore these practical considerations.

Consequently, we propose ***measured*** statistical models of spread spectrum under fading conditions which include practical influences associated with the correlation and demodulation processes.

3. DS/SS EXPERIMENTAL RESULTS

Spread spectrum was investigated under diverse conditions to obtain channel models reflecting the practical influences of the correlation and demodulation processes.

We firstly investigated the influence of AWGN noise on the spread spectrum waveform with $PG = 125$ and $PG = 64$ (figure 1). As expected, the model of figure 2, reduces to the degenerate case; one error-free state and one error state. If we assign the measured BER to p and define q as $1 - p$, we have

$$P(0^m/1) = q^m \tag{6}$$

and the model reduces to

$$p_{11} = p_{21} = q \text{ and } p_{12} = p_{22} = p \tag{7}$$

with transition matrix

$$\begin{bmatrix} q & p \\ q & p \end{bmatrix}$$

It thus means that no matter which state it is in, there is always a probability q of being succeeded by an error-free bit, and a probability p of being succeeded by an error.

In assessing channel models for spread spectrum under multi- user conditions, a graph as expressed by figure 4, was obtained. One user was assigned the "desired user" and indicated as P_D; the other was assigned the "undesired user", indicted by P_U, with $PG = 125$. This graph agrees with equations reported in [4] and [7].

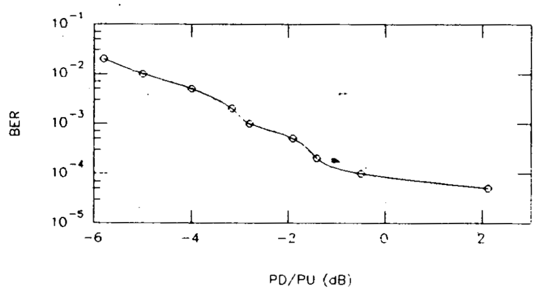

Figure 4: BER against varying user power

The channel model again reduces to a two state model with transition probabilities as described above for the AWGN case. Figure 5 depicts a graph of the BER against SNR with $P_D = P_U$. Also displayed on the graph is a simulation of the channel model. It can be seen that the model simulates the channel almost identical.

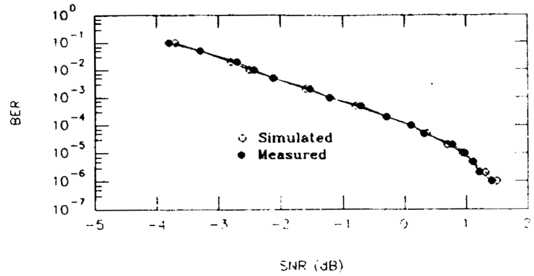

Figure 5: Probability of error for $P_D = P_U$

The most interesting channel model is obtained when multi-path is added to the channel. A three-state partitioned model with two error-free states and one error state is proposed. The amount of fading to obtain a model as described is illustrated in figure 6. As can be seen from the figure, frequency non-selective, flat fading resulted in this model. This type of fading is frequently encountered in outdoor PCN channels [4].

Table 1 and 2 represents the parameters of the model as a function of the average SNR with values as determined by the experimental set up and, respectively, $PG = 125$ and $PG = 64$. The stationary state probabilities of the channel can be represented by

$$P_1 = \frac{p_{31}}{p_{13}} P_3 \tag{8}$$

$$P_2 = \frac{p_{32}}{p_{23}} P_3 \tag{9}$$

$$P_3 = \left(\frac{p_{31}}{p_{13}} + \frac{p_{32}}{p_{23}} + 1 \right)^{-1} \tag{10}$$

4. CONCLUSIONS

Experimental results and mathematical channel models were presented to simulate Direct Sequence Spread Spectrum systems in AWGN, multi-user and multipath conditions, and the validity of the models verified. These models allow us to model Spread Spectrum systems under more realistic conditions.

Partitioned Fritchman renewal Markov processes has been used with great success to model Rayleigh fading channels with AWGN. This paper illustrated that under some conditions of fadin, these type of models can be used for spread spectrum. Further, error distributions

1.2 Partitioned Markov Chain Models

To accurately model a channel, statistical knowledge about the errors to be expected from the channel need to be collected. Fritchman [2] proposed a partitioned Markov chain model with transition probabilities between the states related to the measured channel statistics, yielding both accuracy and simplicity.

The proposed model can briefly be described as follows. Two types of states are postulated; error-free states and error states.

A simple partitioned Markov chain model, in which there is only one error state and no transitions between the error-free states, is shown by Fritchman to be uniquely determined by the *Error-Free Run* distribution. (Partitioned Markov chain models with more than one error state are described in [9]). The transition matrix and state diagram, figure 2, for a model with one error state are displayed below.

$$\begin{bmatrix} p_{11} & \cdots & 0 & \cdots & p_{1N} \\ \vdots & p_{22} & 0 & \cdots & p_{2N} \\ 0 & & \ddots & & \vdots \\ \vdots & \cdots & 0 & & \vdots \\ p_{N1} & p_{N2} & \cdots & p_{N,N-1} & p_{NN} \end{bmatrix}$$

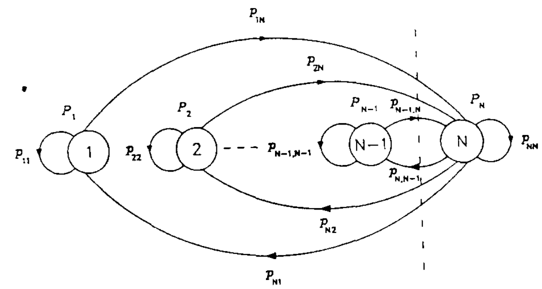

Figure 2: Three State Partitioned Markov State Diagram

The EFR distribution is calculated directly from the measured ***gap distribution***; a gap is defined as the region of error-free bits between two errors, and can be described by a graph of the cumulative relative frequency of the gap length m, versus the length m. The gap distribution gives some indication of the randomness of the channel.

For a random channel with bit-error-rate, p, the probability of a gap length v is $P(0^v) = q^v$, where $q = 1 - p$. The cumulative relative frequency of a gap of length m is

$$p_v(m) = \frac{\sum_{v=1}^{m} q^v}{\lim_{m\to\infty} q^v} = \frac{\frac{q}{1-q}(1-q^m)}{\frac{q}{1-q}} = 1 - q^m \quad (3)$$

The EFR distribution is the probability of an error-free-run of length at least m following an error, $P(0^m/1)$, versus the length m. $P(0^m/1)$ is fitted by a sum of exponential functions of the form

$$A_1 e^{-\alpha_1 m} + A_2 e^{-\alpha_2 m} + \cdots + A_k e^{-\alpha_k m} \quad (4)$$

with the number of terms dictated by the error-free-run distribution and the state transition probabilities, p_{ij} directly related to the constants A_i and α_j [8].

2. EXPERIMENTAL SET UP

The experiments were conducted as depicted in figure 3; data at 16 kbps is differentially encoded and spread by a fast PN- sequence, realizing a processing gain of $PG = 125$ and $PG = 64$. The baseband signal is split into I and Q channels, up converted to 70 MHz, combined and transmitted as a single vector through a HP 3708A noise and interference test set. The interference test set enabled a controlled SNR, ensuring repeatable results. Multi-path was also added to the channel at this point.

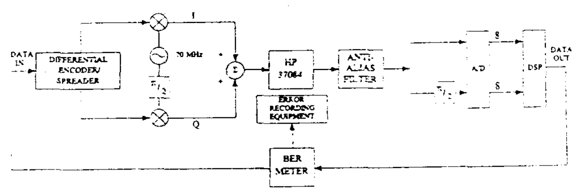

Figure 3: Experimental Set Up

The receiver was realized with DSP techniques to accomplish despreading and demodulation. Correlation was established by a *Digital Matched Filter* (DMF) with transfer function

$$H(f) = kp(f)e^{(-j\omega t_d)} \quad (5)$$

with $p(f)$ the Fourier transform of the received pulse and k an arbitrary constant. Anti-alias filtering of the received signal proved to be of significant importance; inadequate filtering resulted in degradations of SNR as high as 15 dB.

	Transition Probabilities : $PG = 125$									
SNR	p11	p22	p33	p31	p32	p13	p23	P1	P2	P3
-12.4	0.976123	0.77	0.006	0.392	0.60	2.4E-2	0.233	0.817412	0.130652	0.050072
-10.6	0.991017	0.76	0.023	0.389	0.58	8.9E-3	0.241	0.864945	0.048967	0.021417
-10.1	0.998106	0.75	0.126	0.352	0.52	1.9E-3	0.247	0.880273	0.010524	0.005578
-9.1	0.999122	0.75	0.181	0.332	0.48	8.8E-4	0.246	0.884284	0.003939	0.002629
-8.1	0.999845	0.72	0.275	0.312	0.41	1.55E-5	0.278	0.984138	0.000739	0.000496
-7.1	0.999931	0.71	0.322	0.291	0.38	6.9E-5	0.295	0.992187	0.000267	0.00024
-6.1	0.999995	0.67	0.390	0.267	0.34	5E-06	0.326	0.992045	0.000021	0.000021
-5.7	0.999999	0.59	0.486	0.215	0.29	1E-06	0.407	0.996024	3.7E-06	5.5E-06

Table 1: Model Parameters for $PG = 125$ against Signal-To-Noise Ratio

	Transition Probabilities : $PG = 64$									
SNR	p11	p22	p33	p31	p32	p13	p23	P1	P2	P3
-9.5	0.976037	0.76	0.099	0.423	0.471	2.4E-2	0.238	0.881452	0.100106	0.048461
-7.7	0.991112	0.75	0.156	0.401	0.442	8.8E-3	0.248	0.891311	0.035561	0.021111
-7.3	0.998113	0.74	0.202	0.397	0.401	1.8E-3	0.250	0.992213	0.007998	0.004971
-6.2	0.999112	0.74	0.240	0.367	0.391	8.8E-4	0.252	0.835134	0.003096	0.002378
-5.2	0.999825	0.71	0.266	0.348	0.384	1.75E-4	0.287	0.996134	0.000669	0.000501
-4.2	0.999931	0.70	0.323	0.312	0.363	6.9E-5	0.297	0.893631	0.000244	0.000224
-3.2	0.999994	0.69	0.362	0.296	0.341	6E-06	0.304	1.078934	0.000022	0.000019
-2.8	0.999999	0.65	0.432	0.254	0.312	1E-06	0.347	1.080166	4.5E-06	4.6E-06

Table 2: Model Parameters for $PG = 64$ against Signal-To-Noise Ratio

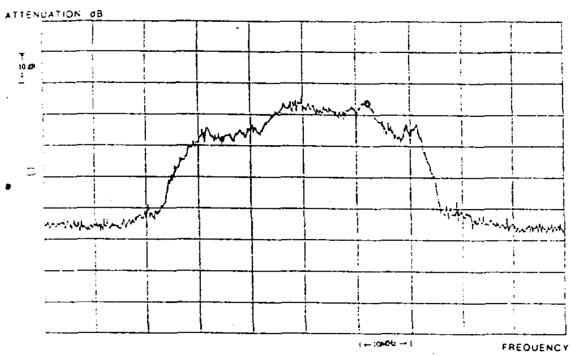

Figure 6: Spectra of Spread Spectrum Under Fading and AWGN Conditions

such as the probability of r errors occurring in s channel bits can be obtained analytically and directly from the Markov model to evaluate the performance of error-correcting codes [8].

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