CAPACITY EVALUATION OF SPREAD SPECTRUM MULTIPLE ACCESS

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Abstract — This paper investigates the capacity of spread spectrum multiple access (SSMA) in a non-cellular environment under additive white Gaussian noise (AWGN) conditions. A theoretical system model is proposed that allows one to evaluate such a system. The probability density function of the average degradation due to the interuser interference is determined by simulations and the validity of approximating the interuser interference as a Gaussian random variable is verified. The capacity of SSMA is shown to be low compared to TDMA when operated in a non-cellular environment without forward error control (FEC) coding.

I Introduction

There has been recent interest in the use of direct sequences (DS) spread spectrum multiple access (SSMA) for use in indoor wireless local area networks and rural communications in the 2300 MHz - 2400 Mhz frequency range [2] [1]. Some of the benefits of a DS SSMA system over TDMA architecture are that it is asynchronous [7] and possesses inherent diversity to multipath and is robust to time varying channels. Furthermore, SSMA allows frequency reuse if a star network is employed, and in addition provides build in addressing and security.

Disadvantages of employing a DS spread spectrum system include: the near/far problem, interference to existing systems (if frequency overlays are employed), and the inferior probability of error performance possible for a given number of users.

In contrast to the more conventional methods of multiple access such as TDMA and FDMA, SSMA does not have any sharply defined system capacity. As the number of users increases, the signal-to-interference ratio becomes smaller and there is a gradual degradation in performance until the SNR falls below threshold. Thus the system can tolerate significant amounts of overload if the users are willing to tolerate poorer performance. An important consideration in SSMA is the number of users that can be accommodated simultaneously. This problem is addressed in a subsequent section. A more detailed discussion of SSMA and spread spectrum techniques in general can be found in Pickholtz [6].

In the section that follows a system model and notation is defined that is used throughout and is based on that developed by Pursley and utilized by Kavehrad in analysis of SSMA for indoor wireless radio [3].

Expressions signifying the capacity of SSMA, with BPSK and DPSK as modulation scheme, is derived for AWGN conditions and is presented in Section III. These expressions are comparable to expressions derived in Pursley [5] for uncoded binary SSMA.

In Section IV we comment on the Gaussian noise approximation commonly used to simplify analysis of the interuser interference in SSMA systems.

We conclude the paper in Section V by summarizing the results and observations.

II System Model

The general architecture of the system that will be analyzed is illustrated in Figure 1, which has been used frequently in previous studies of asynchronous SSMA systems [3].

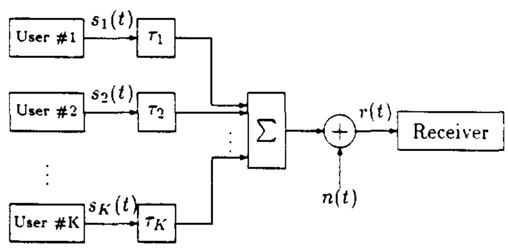

Figure 1: General system architecture

The system consists of K active users transmitting asynchronously. Each user transmits using a different spreading code. The signal transmitted by the kth user, $s_k(t)$, is assumed to be delayed by τ_k, which is uniformly distributed over $[0, T)$ with a random carrier phase θ_k, uniformly distributed over $[0, 2\pi)$. Thus if the receiver of the ith user is attempting to receive the signal transmitted by the jth user, the demodulated signal will consist of the desired signal and interference due to combination of the AWGN and the cross- correlation from the signals transmitted by the other users on the system. The notation, in complex form, to analyze the performance of such a system will now be developed.

A Definition of notation

Assume a SSMA system consisting of K users each utilizing a PN sequence of length N_i chips, which spans one data symbol. Define the kth user's complex baseband information signal as

$$x_k(t) = \sum_{p=-\infty}^{\infty} x_{pk} P_T(t - pT) \qquad (1)$$

in which T is the baud period and $P_T(\cdot)$ is a rectangular pulse of T seconds duration. The term x_{pk} is the complex baseband symbol of the kth user during the pth symbol period, defined by

$$x_{pk} = b_{pk} e^{j\theta_{pk}} \qquad (2)$$

in which b_{pk} and θ_{pk} are the amplitude and phase of the complex baseband output, and $j = \sqrt{-1}$. Similarly to the definition of $x_k(t)$, the DS spreading chip waveform is defined as

$$a_k(t) = \sum_{m=-\infty}^{\infty} a_{mk} \psi(t - mT_c) \qquad (3)$$

in which a_{mk} is the mth chip of the kth user, $\psi(\cdot)$ is the chip waveform, and T_c is the duration of the chip pulse width $N_c = \frac{T}{T_c}$. In the above and subsequent definitions, the first subscript refers to the user under consideration, whereas the second subscript denotes the time interval under consideration (i.e., a particular symbol or chip). The sequence $a_k(t)$ is multiplied with $x_k(t)$ (as was mentioned previously the data and chip symbol transitions are assumed to be aligned), and the resulting sequence is used to bi-phase modulate a carrier at frequency f_c. The resulting transmitted signal of the kth user is

$$s_k(t) = \sqrt{\frac{2E_s}{T}} Re\left\{\xi_k(t) e^{j(\omega_c t + \phi_k)}\right\} \qquad (4)$$

in which $\xi_k(t)$ is the complex baseband signal of the kth user, defined by

$$\xi_k(t) = a_k(t) x_k(t) \qquad (5)$$

and ϕ_k is the random phase of the kth carrier. The binary PN sequence is transmitted using the signal point x_{pk} and the signal point antipodal to x_{pk}. Furthermore, E_s is the symbol energy defined by

$$E_s = \int_0^T [s_k(t)]^2 dt \qquad (6)$$

Assuming an AWGN channel, the received signal may be expressed as

$$r(t) = \sqrt{\frac{2E_s}{T}} Re\left\{\sum_{k=1}^{K} \xi_k(t - \tau_k) e^{j(\omega_c t + \beta_k)}\right\} + n_n(t) \qquad (7)$$

in which τ_k is the random delay of each user's signal arriving at the receiver, and

$$\beta_k = \phi_k - \omega_c \tau_k \qquad (8)$$

is the random phase of the kth user's carrier at the receiver. Furthermore, $n_n(t)$ is a sample of the AWGN with one-sided spectral density $\sigma_n^2 = N_0$.

For the purpose of analysis the receiver for the ith user can be implemented as a correlator matched to the carrier

$$\Phi_i = \sqrt{\frac{2}{T}} a_i(t) cos\omega_c t \qquad (9)$$

which has the form of a conventional BPSK demodulator/correlator matched to the ith user's PN sequence. This receiver is optimum in an AWGN environment, but not in a SSMA environment due to the presence of interuser interference.

In complex notation, the receiver performs an operation equivalent to correlating $r(t)$ with $\sqrt{\frac{2}{T}} a_i(t) e^{-j\omega_c t}$. Defining $Y(t)$ to be the equivalent complex low-pass output of the correlation process gives

$$\begin{aligned} Y(t) &= \sqrt{\frac{2}{T}} \int_0^T r(t) a_i(t) e^{-j\omega_c t} dt \qquad (10) \\ &= 2\frac{\sqrt{E_s}}{T} \int_0^T \{\vartheta(t) + n_n(t)\}\, a_i(t) e^{-j\omega_c t} dt \end{aligned}$$

where $\vartheta(t) = Re\left\{\sum_{k=1}^{K} \xi_k(t - \tau_k) e^{j(\omega_c t + \beta_k)}\right\}$.

The reference sequence is equal to the signal $a_i(t)$ if the receiver is intended to receive that particular signal and if the reference is properly synchronized. The integration carried out by (10) on (7) in an AWGN channel, averages the product of the reference and the incoming signals over T. Assuming without loss of generality that $\tau_i = 0$ and $\beta_i = 0$. The output of this integrator, at some arbitrary baud interval p, can be represented as

$$Y_{pi} = \sqrt{E_s}\left\{x_{pi} + \frac{1}{T}\sum_{k=1}^{K} \rho_{ik} e^{j\beta_k}\right\} + \eta_{pi}, \qquad (11)$$

which is valid if one assumes that $\omega_i \gg T^{-1}$. The normalized correlation coefficient, ρ_{ik}, between $a_i(t)$ and $a_j(t)$ and η_{pi} is the AWGN term associated with $a_i(t)$.

Equation (11) enables us to derive an expression to determine the capacity of SSMA, which is the subject of the next section.

III Capacity of SSMA

For SSMA employing binary signalling, the PSK waveforms are antipodal and, hence, the probability of error with a matched filter receiver is given by

$$P_b = \frac{1}{2} erfc(\delta_i) \qquad (12)$$

for BPSK and

$$P_b = \frac{1}{2} e^{-\frac{1}{2}\delta_i} \qquad (13)$$

for DPSK, where δ_i is the SNR of the ith user. By calculating the SNR for a SSMA system, and substituting in (12) and (13) we can derive expressions for the probability of error.

Following Pursley [5] and Turin [8] we can write Y_{pi} in short as

$$Y_{pi} = \sqrt{E_s}\{x_{pi} + \eta_{ij}\} + \eta_{pi} \qquad (14)$$

where η_{ij} is the multiple access noise, described by an AWGN process with zero mean (more on this in the following section) and η_{ij} a sample of white noise. The output signal-to-noise plus interference ratio for the ith signal may now be written as

$$\delta_i = \frac{\{\sqrt{E_s} x_{pi}\}^2}{E\left\{[\eta_{ij} + \eta_{pi}]^2\right\}} \qquad (15)$$

where $E\{x\}$ signifies the expected value of x. From (15) and the fact that $x_{pi} = \pm 1$, $(x_{pi}^2 = 1)$ we have

$$\delta_i = \frac{E_s}{E\{\eta_{ij}^2\} + 2E\{\eta_{ij}\eta_{pi}\} + E\{\eta_{pi}^2\}} \qquad (16)$$

Since the noise is white with zero mean, $E\{\eta_{ij}\eta_{pi}\} = 0$ and $E\{\eta_{pi}^2\} = \frac{N_0}{2E_s}$. From [8] we have an approximation of the variance for multiple access noise, given by

$$\sigma_{ma}^2 = E\{\eta_{ij}^2\} \approx \frac{K-1}{3N_c} \qquad (17)$$

which enables us to write the final (approximate) expression for the signal-to-noise ratio as

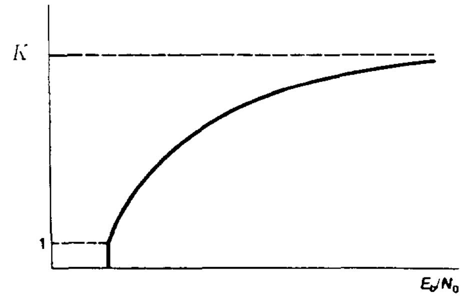

Figure 2: Number of active users in a multiple access system

$$\delta_i \approx \left[\frac{K-1}{3N_c} + \frac{N_0}{2E_s}\right]^{-1} \qquad (18)$$

From the above expression it is evident that for $K = 1$ the signal-to-noise is the same as for a narrow band system. We can now proceed to investigate two scenarios to be encountered by a SSMA system; all users transmitting with equal power and users transmitting at different power levels.

To find the number of users that can be active simultaneously when the received powers from all the users are the same, we can proceed by solving equation (18) for K, the number of active users, to yield

$$K = 1 + \left[3N_c\left\{\frac{1}{\delta_0} - \frac{1}{\delta_1}\right\}\right] \qquad (19)$$

with δ_0 the desired output SNR and $\delta_1 = \frac{3N_c}{N_0}$, where $\delta_1 > \delta_0$. (The integer part of x is signified by $[x]$.)

The variation of the number of active users K with the parameter $\frac{E_s}{N_0}$ is illustrated in Figure 2.

As $\frac{E_s}{N_0}$ becomes large, the number of users asymptotically approaches an upper bound that depends upon the processing gain and the desired output signal-to-noise ratio. It should be remembered, however, that this maximum number of users is a maximum only in the sense that more users would result in a decreased value of output SNR. If one were willing to tolerate a smaller value of output SNR, the number of users could be increased still further.

As an example of the computation of K, consider a system with the following parameters: $\delta_0 = 14$, $N_c = 28dB$ and $\frac{2E_s}{N_0} = 25$. Using equation (19), the maximum number of users, therefore, becomes 42. It should also be noted that by doubling the number of users, would roughly have the effect of reducing the output SNR by 3 dB.

By substituting (18) in (13) and (14) we can calculate the probability of error of a SSMA system. Since DPSK is frequently used in SSMA, Table 1 gives an indication of the capacity with a realistic value of $\frac{E_b}{N_0}$ of 15dB. We can see that the maximum number of equal-power users that can be accommodated by a well-functioning DPSK/SSMA DS system, without fading, is 10-20 percent of the processing gain.

P_b	$\frac{E_b}{N_0} = 15dB$
10^{-3}	$0.19N_c$
10^{-4}	$0.12N_c$
10^{-5}	$0.09N_c$

Table 1: Probability of error for the maximum number of users in a SSMA system

To illustrate the near/far effect, we now consider the number of active users with unequal powers. Since there are many ways in which the received powers can be unequal, it is necessary to specify a particular situation in order to compute specific results. For this purpose assume that all users transmit with equal powers, but are different distances from the ith receiver. We can rewrite (18) as

$$\delta_0 = \frac{3N_cP_i}{\frac{N_0}{T_c} + \sum_{j=1}^{K}\left[\frac{d_i}{d_j}\right]^{\alpha} P_j} \quad (20)$$

where the interuser powers are scaled by the distance d_j and α is the propagation law, normally equal to 4.

Assume that all transmitters are at the same distance from the receiver except for one, that is, $d_j = d_i$ for all $i \neq 1, j$ and $d_1 = \frac{d_i}{2.5}$ with $\alpha = 4$. Calculating the number of users, under the same conditions as was done for the equal power case, leads to a decrease in the number of users from $K = 42$ to $K = 14$. We note that the number of users has been reduced by a factor of 3 simply by virtue of one of the transmitters being 2.5 times closer than all of the others. It might also be noted that if $d_j < \frac{d_i}{2.78}$ this system would fail as a multiple access system since only one user could be supported and none of the others would be able to be received with the desired output SNR.

IV Gaussian approximation

The first term in (18) is the noise contribution due to multiple access noise. By simulating Gold sequences with a random delay τ and carrier phase ω_c the pdf was obtained and depicted in Figure 3.

By fitting the curve in Figure 3 to

$$px(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{\frac{x^2}{2\sigma^2}} \quad (21)$$

it is verified that the multiple access noise is indeed Gaussian with zero mean and variance approximately

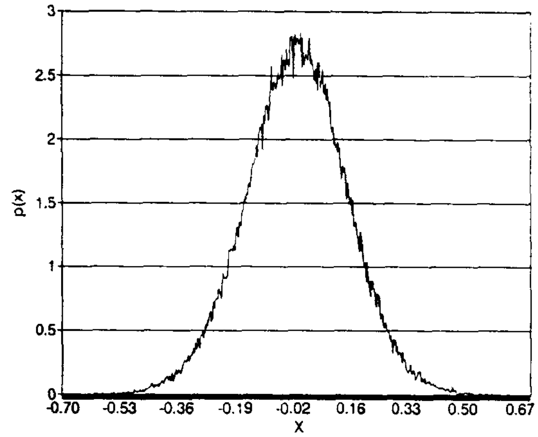

Figure 3: Simulated pdf of multiple access noise

equal to $\frac{K-1}{4N_c}$. The expression derived by Turin [8] is for "coin-flip" codes. (coin-flip codes are random uniformly distributed codes). Since Gold codes have superior correlation properties we find that the variance for these codes are lower.

V Conclusions

We derived an expression to evaluate the capacity of SSMA. When comparing these results with conventional multiplexing techniques the use of spread spectrum seems very inefficient. However, by employing low rate FEC codes we can severely increase the capacity of SSMA. In a band limited environment, the use of FEC codes would result in a penalty being paid for in bandwidth. In SSMA, however, FEC codes can be employed with no resulting expansion in bandwidth or decrease in processing gain. Further, the capacity of SSMA can be enhanced by a system architecture supporting the cellular concept; by reusing the same frequency in all adjacent cells and sectorizing each cell, the capacity of SSMA can exceed that of TDMA by as much as 50% [4]. As was seen the near/far effect reduces the capacity dramatically. In mobile systems dynamic power control has to be employed, while in stationary systems the stations need to be adjusted so that the received power of all users are equal at a particular receiver. It is thus seen that the penalty paid for greater capacity in SSMA is a higher overall system complexity.

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