COHERENT PSK PERFORMANCE OF A CELLULAR DS-SSMA SYSTEM WITH CODING

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Abstract. In this work we consider the application of channel coding to cellular DS-SSMA digital radio. A simple, yet effective, model is proposed to evaluate cellular system performance and capacity. Cellular system performance is further investigated with voice activity monitoring and cell splitting under AWGN, Rayleigh fading and multipath conditions typical for personal and mobile communications. Block- and convolutional codes of relative low complexity are investigated when Gold sequences of length 511 and 255 are employed for purposes of PN spreading. The average degradation due to interuser interference is determined by employing exact aperiodic correlation parameters as defined by Pursley [1]. Numerical results indicate that for a given processing gain and number of interfering users, appropriate coding can allow for reliable communication even under multipath fading conditions.

Key Words. Direct Sequence Spread Spectrum; Error Control Coding; Cellular Communications

1. INTRODUCTION

Spread spectrum signalling techniques, with their inherent anti- multipath, multiple access and rejection of interference capabilities have increasingly received attention for cellular personal and mobile communications. Until recently the standard analysis of SSMA systems was rather pessimistic about the capacity of these systems compared to FDMA and TDMA. Gilhousen et al [2] recognized that since SSMA capacity is only interference limited (unlike FDMA and TDMA) any reduction in interference converts directly and linearly into an increase in capacity. Therefore, by employing a voice activity factor, sectorizing the cells and using various forms of diversity it is possible to achieve SSMA system capacity at least as good as FDMA and TDMA. This improvement has been indicated by [3] and others under AWGN conditions.

In this paper we give a detailed analysis of the performance of a block and convolutional coded cellular SSMA system under frequency-selective slowly fading Rayleigh and multipath conditions which are typical to the indoor wireless channel. For benchmark purposes we present the achievement of a coded SSMA system under AWGN conditions.

There is a sizable literature relating to the effects of multiple access interference on the performance of cellular DS-SSMA, among which are [4] and [5]. Yung [4] considered a cellular SSMA system under Rayleigh fading conditions, modelling the multiuser interference as Gaussian noise (as formulated by Pursley [1]). We also make use of the Gaussain assumption, but calculate exact correlation parameters under fading and multipath conditions. This is accomplished by a family of balanced Gold sequences, of length 511 and 255, with correlation parameters as defined by Sarwarte and Pursley [6].

Using the equations derived in this work it is possible to predict DS/SSMA capacity under the mentioned conditions. We do not, however, set out to predict system capacity, but rather give some comparative results and show that simple block- and convolutional coding can allow for reliable communication under fading conditions. By introducing a voice activity factor of 3/8 and cell splitting of 3, the performance of a 7 cell system with block coding are assessed. We assume hard decisions are made by the demodulator and that the error-producing mechanism results in independent error events or that fading would not cause more than t errors in a block of n bits in the block coded case. The latter assumption requires interleaving at the transmitter and de-interleaving at the receiver.

Section 2 analyses and describes our proposed cellular system model, typical for an indoor wireless communication channel. Numerical results are presented and discussed in Section 3. Finally, a summary and conclusions are presented in the last section of this paper.

2. MODEL AND ANALYSIS

The model considered will be summarized briefly and is based on the non-cellular model developed by Kavehrad [5].

Measurements by Qualcomm [7] indicate that the inter- and intracell interference contribute approximately 50% of the total interference (for equally loaded cells). Therefore, making use of this assumption, the maximum number of users a cellular system can support is given by

$$K' = K\frac{V_{on}}{N_{sect}}1.5 \tag{1}$$

where K is the total number of multiple access users per cell, V_{on} the voice activity factor and N_{sect} the cell splitting factor. Consequently $\frac{N_{sect}}{V_{on}} = 8$.

Our conjectural system will assume $\frac{N_{sect}}{V_{on}} = 8$ and $K = 10$ for a total of approximately 58 users/cell.

An equivalent spread spectrum multipath system model for K users is indicated in Figure 1. The channel for the desired transmitter and receiver ($k = 1$) can be represented by an L- paths Rayleigh fading model where a single transmitted pulse is received via L-paths at the random instant $t_l, l = 1, \cdots, L$. We assume t_l is uniformly distributed over one bit period $(0, T)$ and that each user's code sequence has a period of $N = T/T_c$.

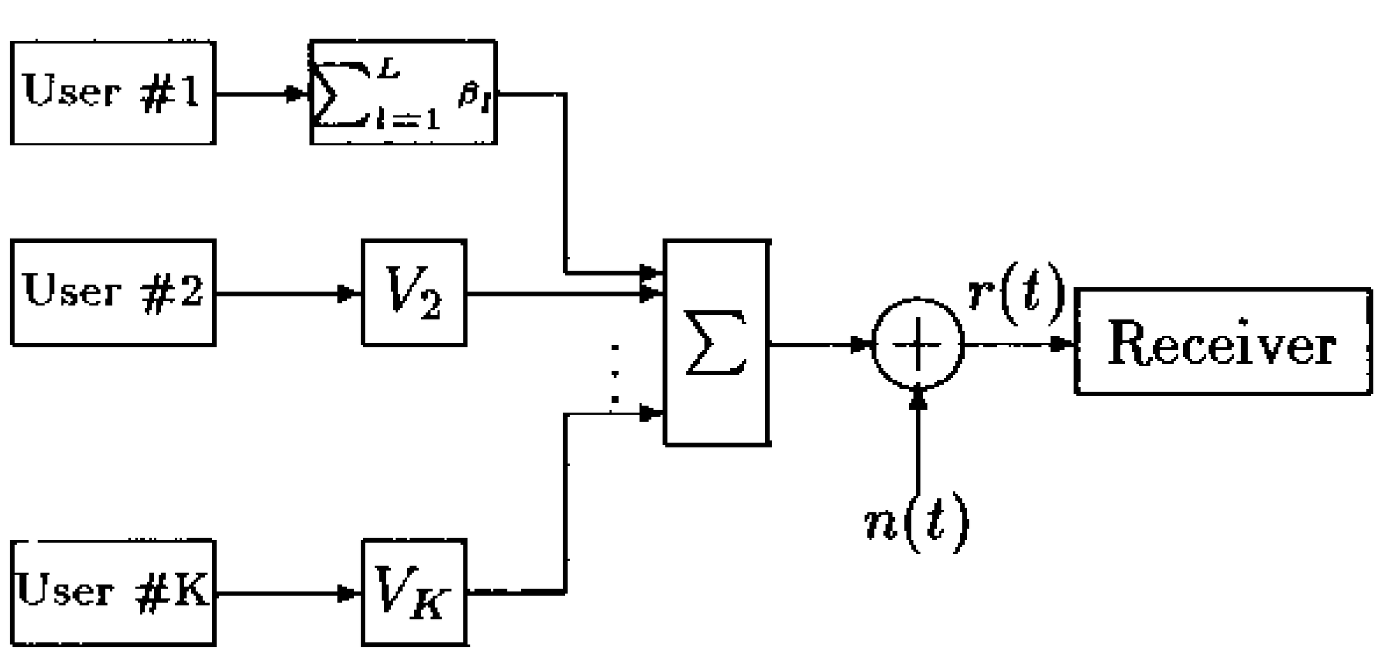

Fig. 1. General system architecture

In the analysis we assume that average power control is assumed which also includes averaging the channel fading characteristics. Baseband signalling at a rate less than the channel coherence bandwidth ensures that intersymbol interference can be neglected. Therefore, the channel has a low-pass equivalent impulse response, given by

$$h(t) = \sum_{l=1}^{L} \beta_l \delta(t - t_l) e^{j\phi_l}, \tag{2}$$

where $\delta(\cdot)$ is the Kronecker delta function, β_l is the Rayleigh distributed path gain and ϕ_l is the random path phase, uniformly distributed between $(0, 2\pi]$.

In the transmission model it is further assumed that the kth interfering user of the multiple access system is linked to the receiver of Figure 1 via a single Rayleigh fading path with a uniformly distributed random delay τ_k ranging from zero to one bit period, T. Since, on average, multipath reduces the signal quality this assumption will render our results conservative.

In our formulation we specify the Rayleigh distributed path gain of the $K - 1$ interfering users by $V_k, k = 2, \cdots, K$. Thus, the received signal for the fading model described is given by

$$\begin{aligned} r(t) = A\sum_{l=1}^{L} \beta_l a_1(t - t_l) b_1(t - t_l). \\ \cos(\omega_c t + \Phi_l) + A\sum_{k=2}^{K} V_k a_k(t - \tau_k). \\ b_k(t - \tau_k)\cos(\omega_c t + \Psi_k) + n(t) \end{aligned} \tag{3}$$

where $\Phi_l = \theta_1 - \omega_c t_l + \phi_l$, $\Psi_k = \theta_k - \omega_c \tau_k$ and θ_k the phase of the kth user. Also, $n(t)$ is white Gaussian noise with double sided spectral density of level $N_0/2$ and θ_1 can be assumed zero with no loss of generality. Since coherent PSK is considered, the receiver is assumed to coherently recover the carrier phase and delay lock to the first arriving desired signal. Following Kavehrad [5] and standard procedures, after matched filter reception, the conditional probability of error is given by

$$P_{e|\beta_1,\alpha} = \frac{1}{2}\text{erfc}\left\{\sqrt{\frac{E_b}{N_0}}[\beta_1 + \alpha]\right\} \tag{4}$$

where

$$\alpha = x + y, \tag{5}$$

$$x = \sum_{l=2}^{L} \frac{\beta_l}{T}[b_{-1}^1 R_{1,1}(t_l) + \hat{R}_{1,1}(t_l)].\cos(\Phi_l),$$

$$y = \sum_{k=2}^{K} \frac{V_k}{T}[b_{-1}^k R_{k,1}(\tau_k) + b_0^k \hat{R}_{k,1}(\tau_k)].\cos(\Psi_k),$$

and b_0^1 represents the information bit being detected and b_{-1}^1 is the preceding bit, which, due to the channel delay spread, affects the detection of b_0^1 received on the first path between the desired transmitter and receiver. The energy/bit and noise spectral density are represented by E_b and N_0 respectively. The parameters β_l and V_k are sample values of a Rayleigh variable. The discrete partial auto- and crosscorrelation functions are given by

$$R_{k,1}(\tau) = A_{n,k,1}T_c + B_{n,k,1}(\tau - nT_c) \tag{6}$$

$$\hat{R}_{k,1}(\tau) = \hat{A}_{n,k,1}T_c + \hat{B}_{n,k,1}(\tau - nT_c) \tag{7}$$

and, together with the variables $A_{n,k,1}, B_{n,k,1}, \hat{A}_{n,k,1}$ and $\hat{B}_{n,k,1}$, defined in [1], enable us to evaluate the system performance for specific code parameters.

The spreading codes used are Gold sequences with generator polynomials 1314553 and 153071 (in octal) for $N = 511$ and $N = 255$ respectively. Initial loadings of these codes were chosen in such a way as to generate ***balanced*** codes (because of their desirable spectral properties) and not necessarily for optimum correlation properties.

Neglecting fading and multipath (4) reduces to

$$P_{e|y} = \frac{1}{2}\text{erfc}\left\{\sqrt{\frac{E_b}{N_0}}[1+y]\right\} \tag{8}$$

with $V_k = 1$ in (3).

There are two ways to remove the conditioning in (4) and (8). The more accurate way is to employ Gauss Quadrature integration [8] by averaging the conditional probability in (4) and (8) over the interference term α and y respectively. Briefly, this is accomplished by evaluating the moments of α and y, which are applied in evaluation of the weights and nodes of the Quadrature Rule. In our analysis we will not use the GQR method, but rather the Gaussian assumption, to be described next, since it has been shown to be sufficiently accurate when error control coding is used [9].

The alternative method is to assume α and y to be Gaussian distributed variables with zero mean and variance σ_{ma}^2 given by the second moment of (34) in [5, pp 1191-1195]. With this assumption, a closed form expression for (4) and (8) can be derived at as

$$P_e = \frac{1}{2}\left\{1 - \sqrt{\frac{\Lambda\gamma_b}{1+\Lambda\gamma_b}}\right\} \tag{9}$$

and

$$P_e = \frac{1}{2}\text{erfc}\left\{\left(\frac{N_0}{E_b} + 2\sigma_{ma}^2\right)^{-\frac{1}{2}}\right\}, \tag{10}$$

respectively, with

$$\frac{1}{\Lambda} = 1 + 2\frac{E_b}{N_0}\sigma_{ma}^2 \tag{11}$$

and

$$\gamma_b = \frac{E_b}{N_0}\beta_1^2. \tag{12}$$

We notice that (10) is similar to an expression derived by Pursley [1] for PSK signalling under AWGN conditions and that in the absence of multiple access interference and a single-path fading of the desired signal (9) simplifies to

$$P_e = \frac{1}{2}\left\{1 - \sqrt{\frac{\gamma_0}{1+\gamma_0}}\right\}, \tag{13}$$

which is the ideal performance of a single-path Rayleigh fading channel [10].

2.1. Channel Coding

Error control coding can be used with great success in SSMA with no penalty paid in bandwidth by the addition of redundancy to the information bits. In this work the block codes considered are the $(n,k,t) = (7,4,1)$ Hamming code and five BCH codes. The BCH codes under investigation are the $(15,7,2),(31,16,3),(63,30,6),(127,64,10)$ and the $(255,123,19)$ codes. These are all approximately rate $R_{cd} = \frac{k}{n} \approx \frac{1}{2}$ codes.

For a block code that corrects t-errors, the bit error probability bound is given by [11]

$$P_b \leq \sum_{i=t+1}^{n} \frac{i+t}{n} \binom{n}{i} P_e^i (1-P_e)^{n-i}, \quad (14)$$

where n is the coded block length and P_e is the average bit error probability of (9) or (10).

All convolutional codes chosen are rate $R_{cd} = \frac{1}{2}$ with constraint lengths $\nu = 2,3,4,6$ and generator polynomials as given in Clark and Cane [12]. (Rate $R_{cd} = \frac{1}{2}$, constraint length $\nu = 6$ VLSI convolutional encoders and decoders are commercially available.)

Performance bounds for convolutional codes are given by

$$P_2(d) = \sum_{k=(d+1)/2}^{d} \binom{d}{k} P_e^k (1-P_e)^{d-k}, \quad (15)$$

for d odd, and

$$P_2(d) = \sum_{k=d/2+1}^{d} \binom{d}{k} P_e^k (1-P_e)^{d-k} + \frac{1}{2} \binom{d}{d/2} P_e^{d/2} (1-P_e)^{d/2}, \quad (16)$$

for d even. Using the union bound we can upper bound the bit error probability as

$$P_b = \sum_{d=d_{free}}^{\infty} a_d P_2(d), \quad (17)$$

where the coefficients $\{a_d\}$ are dependent on the code structure and tabulated by Clark and Cane [12] for the convolutional codes under consideration.

We assume that the PN spreading sequence spans one code symbol. This implies that under the assumption of fixed throughput (i.e. constant data rate), fixed maximum chip rate and fixed complexity, a rate R_{cd} code must employ a PN spreading sequence shorter by a factor R_{cd} than that of the uncoded case. This results in increased interuser interference due to the poorer cross correlation properties of shorter PN sequences. In our case we use PN sequences of $N = 511$ and $N = 255$ for the uncoded case and coded cases respectively.

3. Numerical results

We will present tables that represent the multiple access variance, σ_{ma}^2, under fading and multipath conditions for $N = 511$ and $N = 255$. These values, in conjunction with (9) and (10), allow us to evaluate the average error rate and capacity of a cellular SSMA system simply and efficiently.

We will investigate the performance of three hypothetical SSMA cases, which is described below, and determine the influence of convolutional and block coding to each of these cases.

3.1. Faded Multiple Access Variance

In Tables 1 and 2 the multiple access variance, σ_{ma}^2, is indicated for $N = 511$ with average path strengths of $E\{\beta_i^2\} = -10$ dB and $E\{\beta_i^2\} = -20$ dB, respectively, for all $i = 1,2,\ldots K$, where $E\{\cdot\}$ indicates expected value.

Indicated in Tables 3 and 4 are the multiple access variance for $N = 255$, also with $E\{\beta_i^2\} = -10$ dB and $E\{\beta_i^2\} = -20$ dB, respectively.

Since the variance for fixed K and variable L varies linearly, these tables can be graphically extended and interpolated to determine the variance for a different value of L. The same argument applies for a fixed L and variable K.

3.2. Case study

To assess the performance of a cellular SSMA system we consider three separate cases:
Case 1 - All signals from transmitter to receiver are corrupted by AWGN. That is, the variables β_l, V_k and L, as depicted in Figure 1 and (3), are all unity. We use (10) to calculate the average prob-

L	2	5	10	15	20	25	30
1	6.68E-05	0.000261	0.000589	0.000911	0.001236	0.001573	0.001833
2	0.000154	0.000348	0.000676	0.000998	0.001323	0.00166	0.00192
3	0.00024	0.000435	0.000763	0.001085	0.00141	0.001747	0.002007
4	0.000327	0.000521	0.000849	0.001171	0.001497	0.001834	0.002094
5	0.000414	0.000608	0.000936	0.001258	0.001584	0.001921	0.00218
6	0.000501	0.000695	0.001023	0.001345	0.00167	0.002007	0.002267
7	0.000588	0.000782	0.00111	0.001432	0.001757	0.002094	0.002354
8	0.000675	0.000869	0.001197	0.001519	0.001844	0.002181	0.002441
9	0.000761	0.000955	0.001284	0.001605	0.001931	0.002268	0.002528
10	0.000848	0.001042	0.00137	0.001692	0.002018	0.002355	0.002614

Table 1 Variance of Multipath faded SSMA, $N = 511$, $E\{\beta^2\} = -10$ dB

L	2	5	10	15	20	30	30
1	6.68E-06	2.61E-05	5.89E-05	9.11E-05	0.000124	0.000157	0.000183
2	1.54E-05	3.48E-05	6.76E-05	9.98E-05	0.000132	0.000166	0.000192
3	2.4E-05	4.35E-05	7.63E-05	0.000108	0.000141	0.000175	0.000201
4	3.27E-05	5.21E-05	8.49E-05	0.000117	0.00015	0.000183	0.000209
5	4.14E-05	6.08E-05	9.36E-05	0.000126	0.000158	0.000192	0.000218
6	5.01E-05	6.95E-05	0.000102	0.000134	0.000167	0.000201	0.000227
7	5.88E-05	7.82E-05	0.000111	0.000143	0.000176	0.000209	0.000235
8	6.75E-05	8.69E-05	0.00012	0.000152	0.000184	0.000218	0.000244
9	7.61E-05	9.55E-05	0.000128	0.000161	0.000193	0.000227	0.000253
10	8.48E-05	0.000104	0.000137	0.000169	0.000202	0.000235	0.000261

Table 2 Variance of Multipath faded SSMA, $N = 511$, $E\{\beta^2\} = -20$ dB

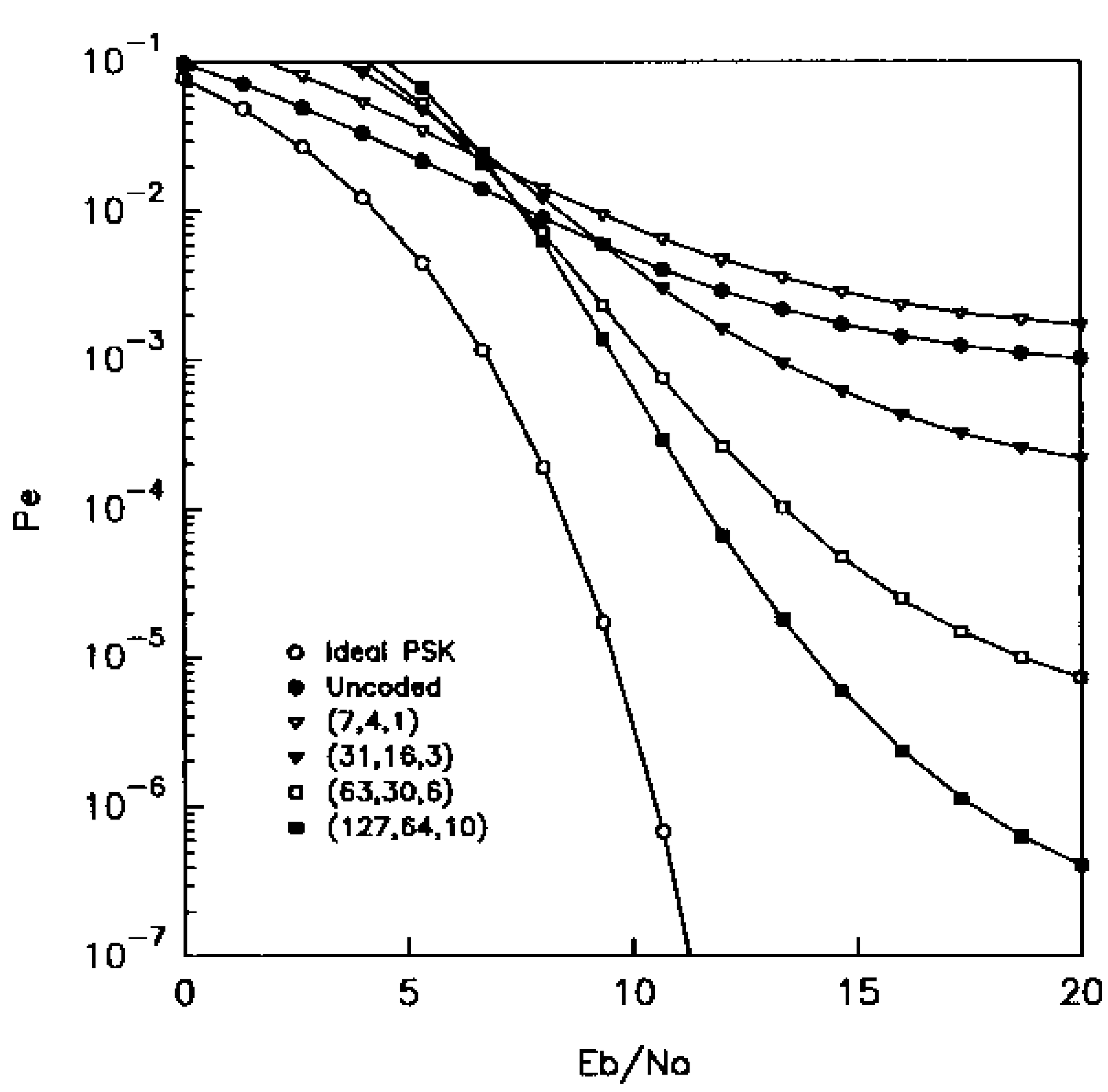

Fig. 2. Average error performance for AWGN channel

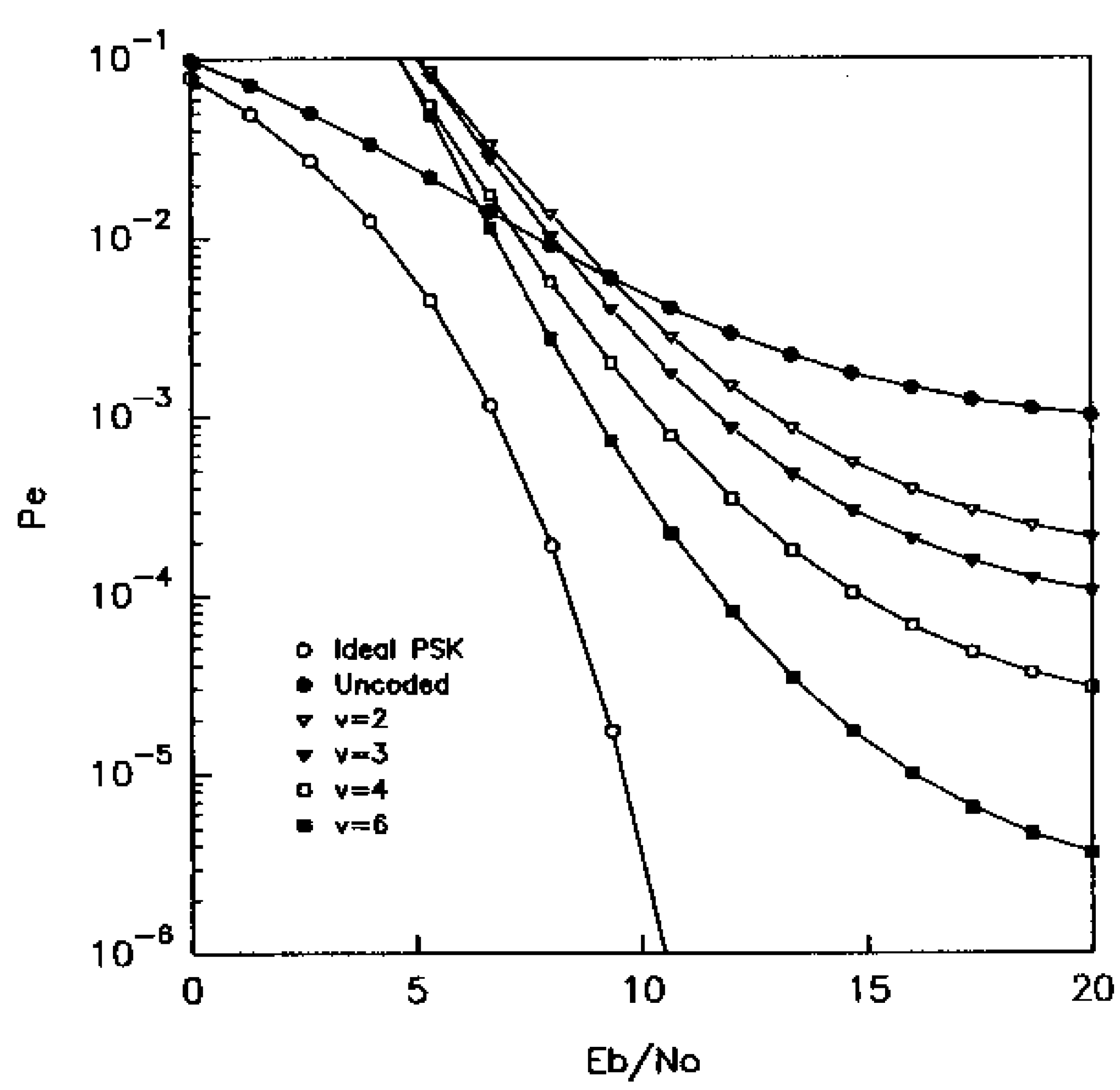

Fig. 3. Average error performance for AWGN channel

L	2	5	10	15	20
1	0.000142	0.000537	0.001156	0.001829	0.002386
2	0.000317	0.000712	0.001331	0.002004	0.002561
3	0.000492	0.000887	0.001506	0.002179	0.002737
4	0.000668	0.001063	0.001682	0.002354	0.002912
5	0.000843	0.001238	0.001857	0.00253	0.003087
6	0.001018	0.001413	0.002032	0.002705	0.003263
7	0.001193	0.001589	0.002208	0.00288	0.003438
8	0.001369	0.001764	0.002383	0.003056	0.003613
9	0.001544	0.001939	0.002558	0.003231	0.003789
10	0.001719	0.002115	0.002733	0.003406	0.003964

Table 3 Variance of Multipath faded SSMA, $N = 255$, $E\{\beta^2\} = -10$ dB

L	2	5	10	15	20
1	1.42E-05	5.37E-05	0.000116	0.000183	0.000239
2	3.17E-05	7.12E-05	0.000133	0.0002	0.000256
3	4.92E-05	8.87E-05	0.000151	0.000218	0.000274
4	6.68E-05	0.000106	0.000168	0.000235	0.000291
5	8.43E-05	0.000124	0.000186	0.000253	0.000309
6	0.000102	0.000141	0.000203	0.000271	0.000326
7	0.000119	0.000159	0.000221	0.000288	0.000344
8	0.000137	0.000176	0.000238	0.000306	0.000361
9	0.000154	0.000194	0.000256	0.000323	0.000379
10	0.000172	0.000211	0.000273	0.000341	0.000396

Table 4 Variance of Multipath faded SSMA, $N = 255$, $E\{\beta^2\} = -20$ dB

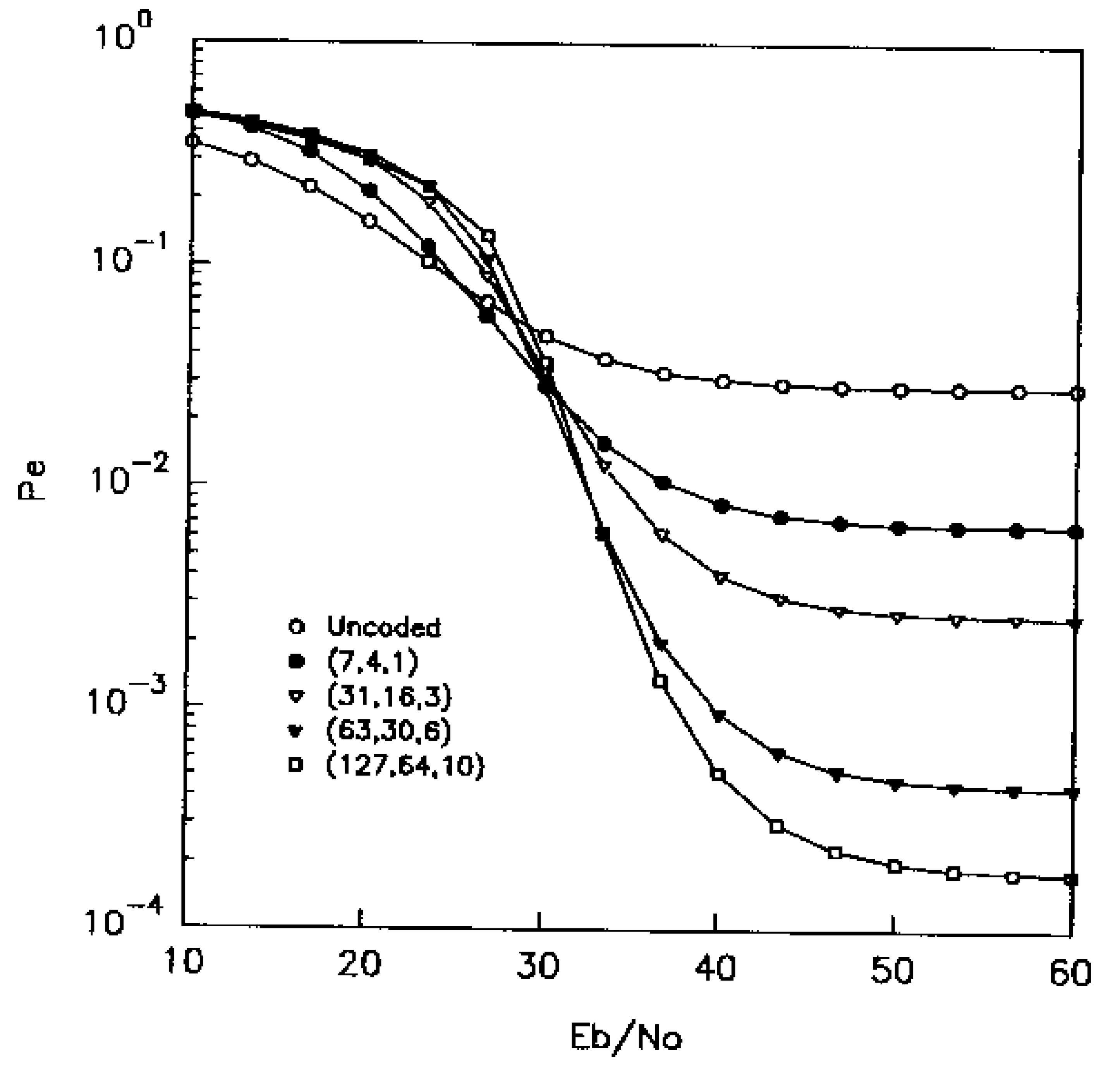

Fig. 4. Block coded error performance under multipath conditions ($L = 1$)

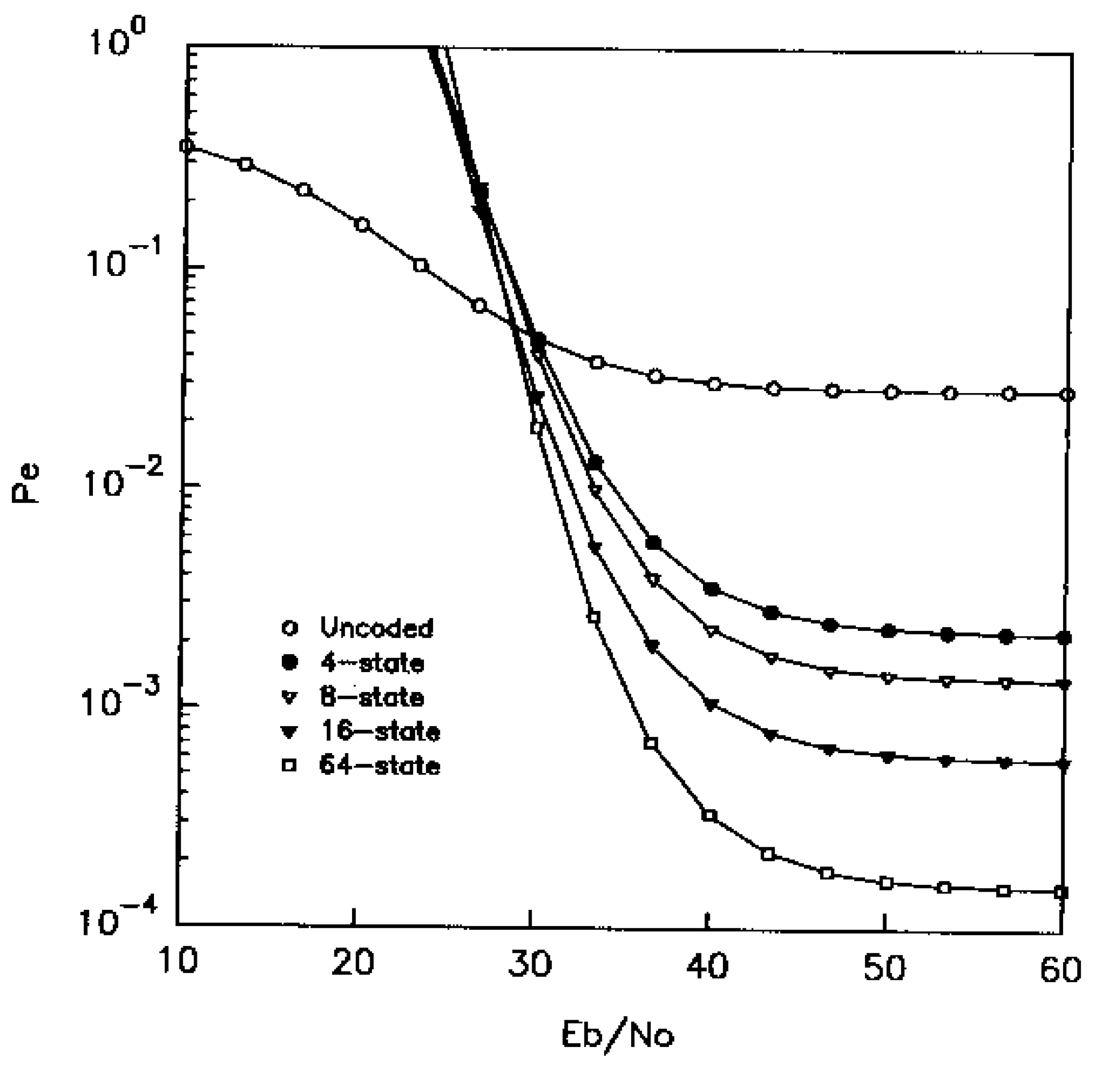

Fig. 5. Convolutional error performance under multipath conditions ($L = 1$)

ability of error. This will serve as benchmark to the faded and multipath cases.
Case 2 - Here we consider the case where $L = 1$, thus eliminating multipath effects on the desired user. All the signals arriving via different paths at the receiver have Rayleigh distributed random gains and therefore σ^2_{ma} is calculated for $L = 1$.
Case 3 - All the *desired* signals arriving via different paths at the receiver together with the interfering signals have Rayleigh distributed random gains. The number of desired paths is limited to ten and σ^2_{ma} in (9) include both fading for the k users and the L paths. This is a scenario in which the transmitter terminals are mobile and gains are Rayleigh with respect to geographic positioning.

In Cases 2 and 3 all average path gains between the desired transmitter and receiver were assumed to be equal. This assumption will result in conservative error probability values for a fixed total interference power.

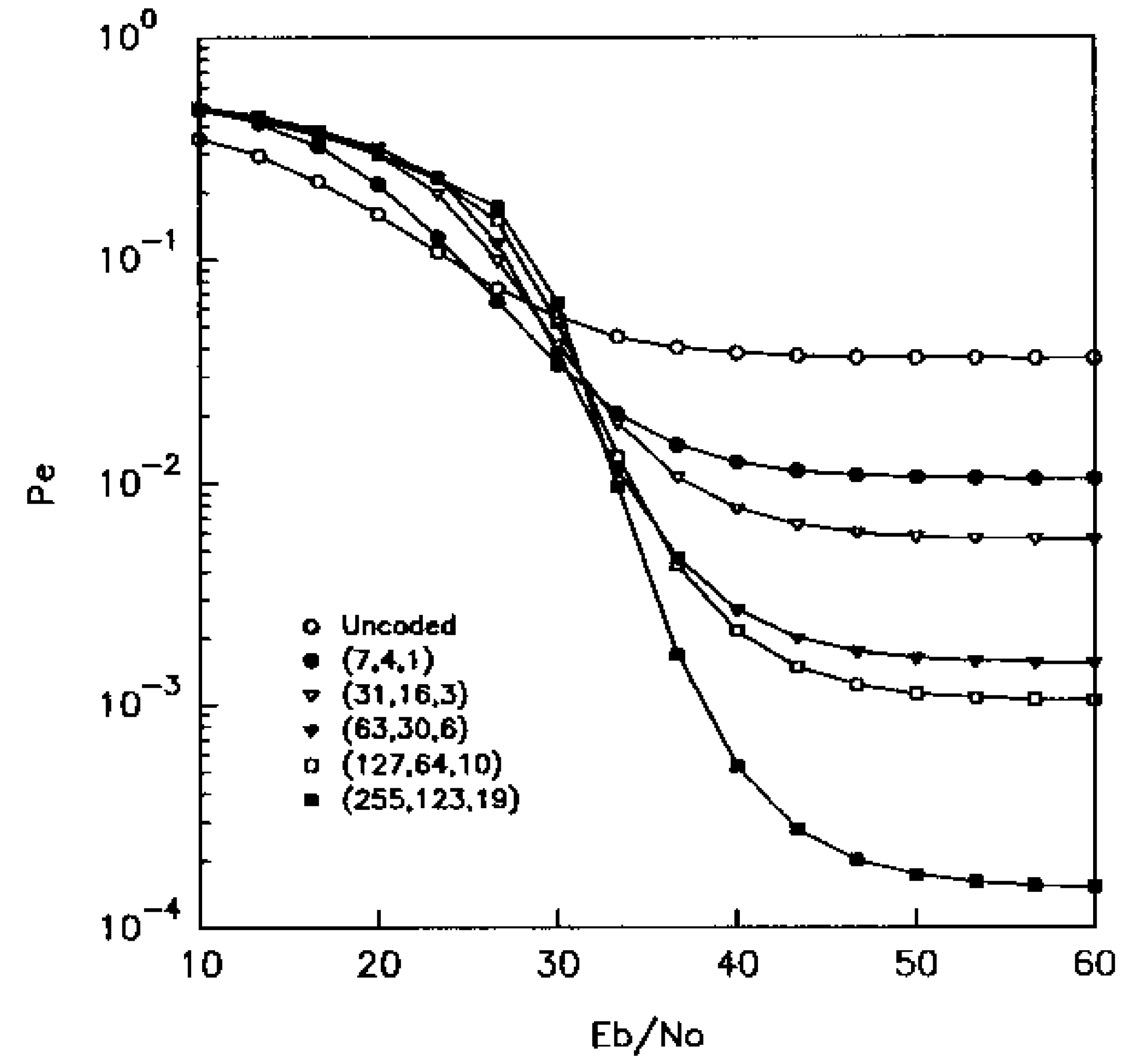

Fig. 6. Block coded error performance under multipath conditions ($L = 10$)

3.3. Detailed Results

Figure 2 and 3 illustrate the block and convolutional coded average error probability as a function of unfaded signal-to-noise ratio, corresponding to Case 1, respectively. In the same figure, performance of an ideal coherent PSK demodulator is shown.

For $K = 150$ (K'=800 users/cell), it is clear that the performance is not acceptable for voice communication. By introducing coding, be that block or convolutional, the situation is improved considerably. It is interesting to note that convolutional codes, with comparable complexity and minimum distance, outperform block codes. Also, convolutional codes are more effective at low signal-to-noise ratios; specifically the $\nu = 6$ case, from $E_b/N_0 > 6$.

Considering Case 2, where the transmitters are mobile and multipath is neglected, the picture looks a lot worse than that of Figures 2 and 3. We assume a hypothetical average path strength of the Rayleigh faded path associated with the kth user to be -20 dB. It is evident that the performance with $K = 10$ (K'=53 users/cell) is totally unacceptable (Figures 4 and 5); the curve saturates at an average error probability of approximately 0.03. By introducing a block code that can correct six or fewer errors, the BCH $(63, 30, 6)$ code in this case, the average error probability saturates at less than 10^{-3}, which is already acceptable for speech (Figure 4). More powerful codes improve the situation even further.

The commercially available $R_{cd} = 1/2, \nu = 6$ convolutional code also improves the error rate sufficiently for reliable communication, as indicated in Figure 5. However, coding, be that block or convolutional coding, is only beneficial at high signal-to-noise ratios, typical 30 dB in this scenario. If lower signal-to-noise ratios are required other forms of diversity has to be considered. Nevertheless, error control alone, without additional forms of diversity, is sufficient to allow for acceptable communication under Rayleigh fading conditions.

From Figures 6 and 7 we see the atrocious performance of the uncoded signal with ten paths for block and convolutional coding respectively. Powerful coding is needed to improve the situation. At least a BCH (127,64,10) is needed to rescue the situation, although the (255,123,19) code would be preferred (Figure 6). From Figure 7 we see that at least the $\nu = 6$ convolutional code is needed to ensure acceptable communication quality. As was the case in the previous situation ($L = 1$), coding only improves the situation at high signal-to-noise ratios, in this case 30 dB.

It is thus evident that error control coding plays an important role in cellular SSMA systems.

4. Summary and Conclusions

The work reported extends previous results in the following respects. Accurate closed form expressions were derived using the Gaussian assumption and an efficient cellular system model was

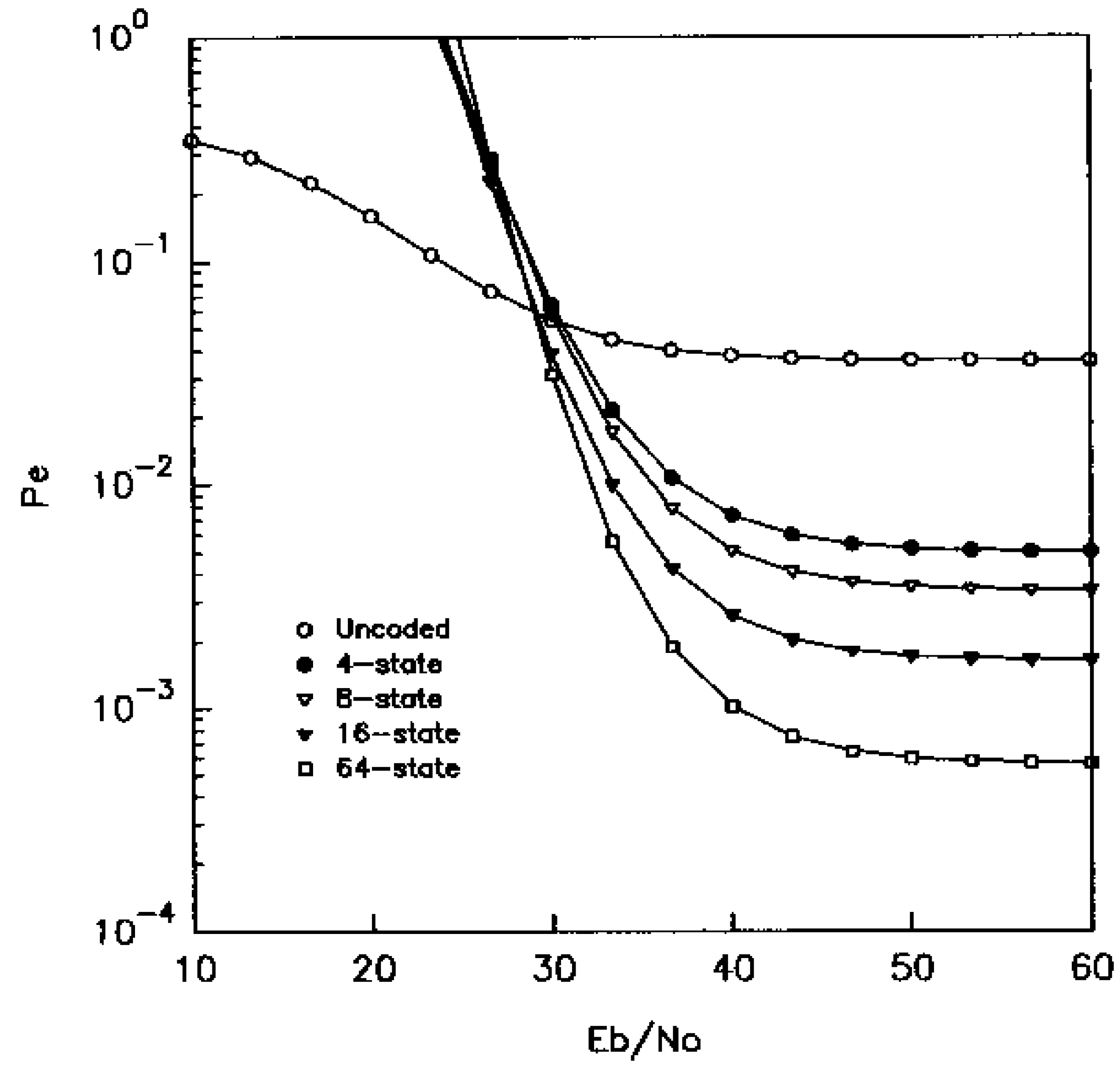

Fig. 7. Convolutional error performance under multipath conditions ($L = 10$)

proposed to evaluate system performance over Rayleigh and multipath fading channels.

From our numerical results we draw the following conclusions:
In an AWGN environment with modest coding, a cellular DS-SSMA system delivers sufficient performance, comparing favourably to TDMA when speech is transmitted. SSMA is not as promising when data is transmitted since the voice activity factor gain can not be utilized. However, coding is very effective when used with SSMA since no bandwidth penalty is paid, making a coded SSMA system very competitive with any of the other multiple access schemes.

When Rayleigh fading and multipath conditions are considered without coding, it seems absolutely necessary to include some form of diversity; otherwise, much higher processing gain is needed to decrease the error probability. When simple block or convolutional codes are used as a form of diversity, the performance is acceptable under these conditions. If better performance is required under fading and multipath conditions, additional forms of diversity and/or more powerful error control codes (with perhaps soft decision) are required.

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