DS-CDMA Performance with Maximum Ratio Combining and Antenna Arrays in Nakagami Multipath Fading

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Abstract — **The Indoor Wireless Communication (IWC) channel is very hostile to a DS-CDMA signal and therefore effective techniques are needed to enhance system performance and capacity. By implementing antenna arrays, diversity schemes or a combination of antenna arrays and diversity techniques, the receiver can discriminate between users by their temporal signatures, spatial signatures and also make use of the statistic independence of the received signal. In this study we are interested in IWC performance and therefore the channel model is assumed to be Nakagami distributed. At the receiver an array of M antennas is used to discriminate between the users based on their spatial diversity. The fading process at each of the antenna elements are statistically dependent and further improvements can be realized by making use of the independent fading characteristics of the received signal. To make use of this statistical independent information, the performance of a P branch Maximum Ratio Combining (MRC) receiver is also considered. We further investigate the performance of a combination of P clusters of M antennas separated by the coherence bandwidth of the channel, thereby making use of both forms of spatial diversity. A comparison of the three schemes (antenna arrays, MRC diversity and a combination of antenna arrays and MRC diversity) under equal complexity conditions are made under multipath fading conditions. It is shown that the performance and capacity of a MRC diversity receiver, under the given assumptions, outperforms the other two methods.**

I. Introduction

In mobile communications, and especially in IWC, fading due to multipath is a major factor in the deterioration of communication quality. In addition, in a DS-CDMA multiple access environment, which is interference limited, the multiple access users contribute to degrade the system performance even further. One strategy to overcome this problem is the adaptive array which eliminates the effect of the delayed waves in a multipath environment and the interfering users by beam-forming or controlling the radiation pattern adaptively. However, in the conventional receiver, the delayed waves are simply eliminated but not utilized. It has been pointed out [1] that the directional reception is effective in reducing the fading and multiple access effects. In ie. [2] and [3] it has been pointed out that the information identical to that in the direct wave is contained in the delayed wave, resulting in an improvement in performance. By adaptively combining the direct and delayed waves (with statistical dependent fading on the direct and delayed waves) the SNR can be improved.

It is further well known that diversity techniques can improve the system performance. By separating the antennas by the coherence bandwidth of the channel, received faded signals are statistically independent.

In this paper, a direct comparison is made between the performance (average error rate) of an antenna array (consisting of M antennas), MRC diversity (consisting of P antennas) and a combination of the two techniques (consisting of $M \times P$ antennas) on a basis of equal complexity for an IWC channel. For a fair comparison the number of antennas is used as criteria. In other words, a $P = 4$ MRC diversity scheme is compared to a $M = 4$ antenna array or a combination of $M \times P = 4$ antennas in a combined diversity and antenna array scheme.

For the particular cases of the Nakagami parameter $m \leq 1$, the fading does not contain a specular component - therefore, in order to demodulate coherently, the implicit assumption is made that some type of carrier recovery is used. This can be achieved via channel-parameter estimation circuits. For the M antenna array elements only one carrier recovery circuit is necessary since the signals have the same characteristics, with only a known phase delay between them. Each of the P diversity branches, however, needs a separate carrier recovery circuit due to the independent fading in each branch.

In our analysis it is assumed that perfect power control is exercised by the base station to keep the power level of the mobiles constant. This restriction can be relaxed by considering a simple successive interference canceler.

The rest of the paper is organized as follows. In the next section the system model is described and a closed form expression to calculate the average probability of error is derived. Sections III and IV respectively describe the numerical results and concludes the paper.

II. System Model and Analysis

A simple cellular system architecture is considered and the analysis is assumed to be for the downlink (basestation-mobiles) - hence power control and synchronization of the PN spreading sequences is easily achieved. In this analysis only intercell interference is considered. M antenna arrays

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are grouped in P clusters. The M antennas represent an antenna array where the received signal is a delayed replica at each of the M antennas, assumed to be within the coherence bandwidth of the channel. Each of the P clusters are separated spatially in such a way that the signals are received statistically independent. When $M = 1$, the parameter $\mathbf{a}_k^H$ is unity. When a combination of antenna arrays and MRC diversity are considered the receiver is termed a Double Spatial and Temporal Filter (DSTF) receiver since it makes use of both spatial diversity techniques and temporal filtering.

In the system there are K users and the data signal, $b_k(t)$, of user k is a sequence of unit amplitude positive and negative rectangular pulses of duration T. The modulation scheme is assumed to be simple coherent BPSK and user $K = 1$ is arbitrarily chosen as the reference user. All the conclusions are nevertheless expected to be generally true for any linear modulation scheme.

The data signal of each user is spread by a spreading sequence, $c_k(t)$, with period $N = T/T_c$. In this paper we used PN sequences with different initial loadings to calculate the numerical results. This implies that some form of code synchronization is needed from the basestation.

The complex lowpass equivalent impulse response of the passband channel for the link between the kth user and the basestation can be represented by

$$h_k(t) = \sum_{l=1}^{L} \beta_l \delta(t - t_l) \exp(j\phi_l), \tag{1}$$

where β_l is a Nakagami distributed random path gain, ϕ_l is the random path phase uniformly distributed between $[0, 2\pi)$. The L paths model stems from the fact that spread spectrum signaling with a transmitted signal bandwidth much wider than the coherence bandwidth of the multipath fading channel enables the multipath components to be resolved.

The transmitted and received signals can respectively be written as

$$\begin{aligned} s_k(t) &= A b_k(t) c_k(t) \cos(\omega_c t + \theta_k) \\ \forall \quad & k = 1, 2, \cdots K \end{aligned} \tag{2}$$

and

$$\begin{aligned} r(t) &= A \sum_{k=1}^{K} \mathbf{a}_k \beta_k c_k(t - \tau_k) b_k(t - \tau_k). \\ & \cos(\omega_c t - \omega_c \tau_k + \theta_k) \\ &+ A \sum_{l=1}^{L} \mathbf{a}_l \beta_l c_l(t - \tau_l) b_l(t - \tau_l). \\ & \cos(\omega_c t - \omega_c \tau_l + \theta_l) + n(t) \end{aligned} \tag{3}$$

with

$$\mathbf{r}(t) = [\mathbf{r_1(t)} \ \ \mathbf{r_2(t)} \ \ \cdots \ \ \mathbf{r_M(t)}]^T \tag{4}$$

and

$$\mathbf{a}_k = [a_{1,k}(t) \ \ a_{2,k}(t) \ \ \cdots \ \ a_{M,k}(t)]^T \tag{5}$$

the array response vector for user k. $a_{m,k}$ is the complex gain from user k to the mth antenna. We also make the assumption that only the reference user's signal is received via multiple paths. Since, on average, multipath reduces the signal-to-noise ratio, this assumption renders our multipath results conservative.

Also in (3), $n(t)$ is white Gaussian noise with double sided spectral density of $N_0T/4$, τ_k the random delay of the kth user and β_k is the Nakagami distributed random path gain of the kth user.

Since coherent demodulation is assumed, the receiver coherently recovers the carrier phase and delay locks to the desired signal. Therefore, after correlation and demodulation, a signal sample at the receiver low-pass filter can be expressed as

$$\xi = \mathrm{Re}\left\{ \int_0^T \frac{\mathbf{a}_1^H}{||\mathbf{a}_1||} \mathbf{r}(t) c_1(t) \cos \omega_c t dt \right\}. \tag{6}$$

By the assumption that the phase delay locking of the receiver to the desired received signal, (6) can be expressed as

$$\begin{aligned} \xi &= \beta_1 \frac{AT}{2} b_0^1 ||\mathbf{a}_1|| \\ &+ \frac{A}{2} \sum_{k=2}^{K} \beta_k \{ b_{-1}^k \mathcal{R}_{k,1}(\tau_k) + b_0^k \hat{\mathcal{R}}_{k,1}(\tau_k) \} \overline{\mathcal{R}}_{k,1} ||\mathbf{a}_k|| \cos \theta_k \\ &+ \frac{A}{2} \sum_{l=2}^{L} \beta_l \{ b_{-1}^1 \mathcal{R}_{1,1}(\tau_l) + b_0^1 \hat{\mathcal{R}}_{1,1}(\tau_l) \} \overline{\mathcal{R}}_{1,1} ||\mathbf{a}_1|| \cos \phi_l \\ &+ \eta \end{aligned} \tag{7}$$

where

$$\overline{\mathcal{R}}_{k,1} = \frac{\mathrm{Re}\left[\mathbf{a}_1^H \mathbf{a}_k\right]}{||\mathbf{a}_1|| ||\mathbf{a}_k||}, \tag{8}$$

b_0^1 represents the information bit detected and b_{-1}^1 is the preceding bit which, due to the channel delay spread, affects the detection of b_0^1 received on the first path between the desired transmitter and receiver.

Using asynchronous correlation properties and the Gaussian Assumption, the average error probability can be derived as

$$P_e = \frac{1}{2}\left\{ 1 - \frac{2\Gamma\left(\epsilon + \frac{1}{2}\right)}{\sqrt{\pi}\Gamma(\epsilon)} \sqrt{\frac{\Lambda\gamma_0}{m}} \, {}_2F_1\left(\frac{1}{2}; \epsilon + \frac{1}{2}; \frac{3}{2}; -\frac{\Lambda\gamma_0}{m}\right) \right\} \tag{9}$$

with

$$\gamma_b = \frac{M E_b}{N_0} \sum_{i=1}^{P} \beta_{1,i}^2 \tag{10}$$

with average

$$\overline{\gamma}_b = \frac{M E_b}{N_0} \sum_{i=1}^{P} E\{\beta_{1,i}^2\} = M P \gamma_0 \tag{11}$$

and P the number of M clustered antennas. The average received SNR expression of (11) can be simplified as shown since each branch is assumed to be perfectly estimated. Also $\epsilon = mP$.

III. Results

Using the results from the previous section, numerical results are presented in this section.

To make a fair comparison it is further assumed that the antenna gains are constant, that the antenna beams are independent and that the antenna spacing of the antenna array is $d = \lambda/2$.

To show the improvement afforded by the antenna arrays, Figure 1 graphically illustrates the results for $m = \infty$ (no fading) with $M = \{1, 2, 3, 4\}$, $N = 31$ and $K = 10$. It is clear that as the number of antennas in the array is increased, the error rate is reduced substantially. The difference in average error rate between $M = 1$ and $M = 2$ is roughly three orders of magnitude. The error rate is reduced from an unacceptable performance of 10^{-3} to almost 10^{-6}. The error curve, however, still shows the irreducible characteristic due to the interference present in the system. The use of interference cancellation techniques should be included to eliminate this effect. From the assumption that the beams are independent and that each user has only one beam, the performance improvement due to the antenna array will start to saturate at $M = K - 1 = 9$. This is due to the fact that an antenna array [4] can only reject a maximum of $M - 1$ interferers.

Figure 2 shows that an increase in the number of antennas is a more efficient way to improve the error performance than to increase the processing gain. To illustrate this point; consider the case where $N = 31$ and $M = 1$. By increasing the processing gain to $N = 511$, the error rate decreases by roughly an order, while increasing the number of antennas to $M = 4$, and keeping the processing gain constant, the error rate decreases by almost two orders of magnitude. This can be attributed to the fact that an antenna array decreases the multiple access interference (ie. tracking the reference user) more than the processing gain increases the SNR.

Under fading conditions, let us first consider the performance with $m = 1$, that is when the Nakagami distribution reduces to the well known Rayleigh fading model. In all the fading results spreading is achieved with PN sequences of length $N = 511$. Figure 3 compares the performance of a $M = 2, P = 1$ ($B = 2$) combination with a $M = 1, P = 2$ ($B = 2$) and a $M = 2, P = 2$ ($B = 4$) combination in a two path environment. Clearly, with an equal number of antennas ($B = 2$), the diversity antenna combination performs substantially better. A combination of the two techniques results in the best performance. This is not a fair comparison since the number of antennas are doubled. From Figures 4, 5 and 6 a better perspective of the combination of the antennas can be gained. Figure 4 shows that as the number of antenna array antennas (M) increase, the performance improves. This performance improvement, however, starts to saturate at $M > 2$ due to the fact that the antenna array can only reject one of the interfering users. On the other hand, from Figure 5, we see that an increase in the number of diversity branches results in a greater performance enhancement - which also tends to saturate as P increases. Figure 6 shows the performance of a combination of the two schemes. For an equal number of antennas, the diversity receiver alone outperforms the DSTF receiver.

Figure 7 shows a direct comparison of all the possible combinations to obtain $B = 6$. It is clear that the performance with a greater number of diversity branches, P, are always better than the performance with an equal number of antenna arrays. In fact, the performance for the combination $P = 2, M = 3$ is better than the performance for $P = 1, M = 6$. Clearly, to improve the performance, under the mentioned system conditions, a larger number of diversity branches is the most efficient in terms of number of antennas used.

Figure 8 shows the capacity for a number of different receiver structure as a function of the Nakagami parameter m and $L = 5$. Low values of m (severe fading) are investigated since this is typical of an IWC channel.

Figure 9 shows the performance of only a diversity receiver, only antenna arrays and a combination of the two techniques as a function of higher values of m. Consistent with previous observations the performance improves as m increases - this is due to less fading as m increases. Similar characteristics, as noted above, regarding the parameter B are noticed as m increases.

IV. Conclusions

The advantage of having a combination of diversity and antenna array antennas is in the flexibility it provides. In an environment absent of multipath fading (where diversity branches are useless) the antenna array elements (M) can be increased to enhance the performance. In a multipath fading environment the number of antenna arrays can be reduced and the number of diversity branches increased. From these results it is clear that making use of the statistically independence of the faded received signals is more important than eliminating the interference in a DS-CDMA system. Also, in an environment where the power of the received signals are unequal, a higher number of antenna arrays would be desired to form the beam onto the desired signal more accurately, thereby reducing some of the high powered interference. Under these circumstances it is expected that the diversity branches will be less effective than the antenna arrays. This analysis is currently underway.

In conclusion it is further noted that the importance of multipath system design parameters can be written as (in order of importance) $P \rightarrow M \rightarrow N$. Therefore, to increase capacity in a DS-CDMA system, it is more desirable to increase either the number of diversity branches (P) or the number of antenna arrays (M) before increasing the processing gain (N). The complexity and bandwidth expansion resulting from an increase in N is normally undesirable.

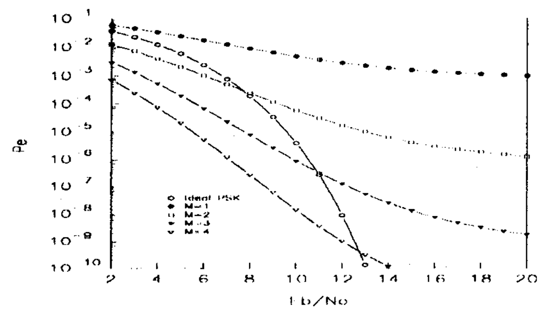

Fig. 1. Performance for $K = 10$, $N = 31$, $M = \{1,2,3,4\}$ and $m = \infty$.

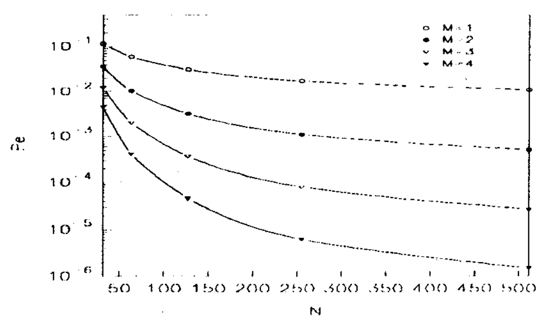

Fig. 2. Influence of the processing gain (N) on system performance; $K = 50$ and $m = \infty$ at $E_b/N_0 = 5$ dB

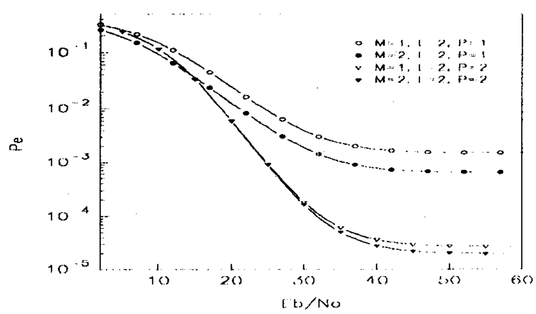

Fig. 3. Comparison of the performance of an equal number of antenna arrays and diversity branches. $K = 2$, $L = 2$, $N = 511$ and $m = 1$

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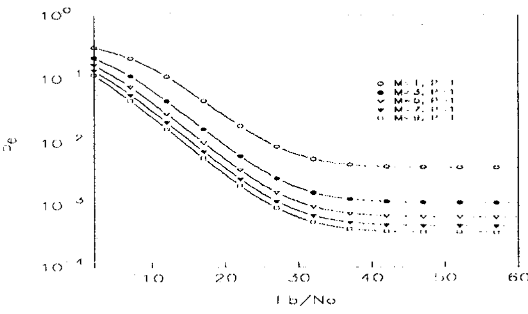

Fig. 4. Influence on the average error rate as the number of antennas in the antenna array increases. $K = 2$, $L = 5$, $N = 511$ and $m = 1$

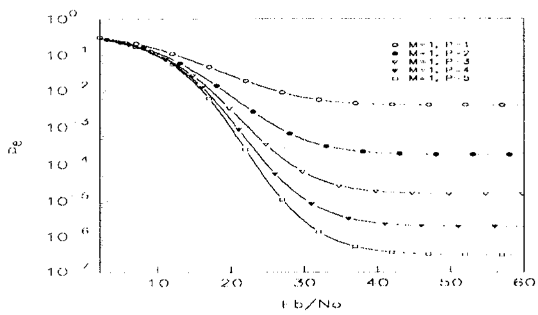

Fig. 5. Influence on the average error rate as the number of diversity branches are increased. $K = 2$, $L = 5$, $N = 511$ and $m = 1$

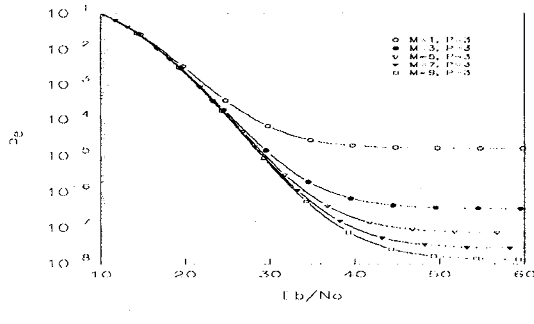

Fig. 6. Performance of the DSTF receiver (combination of antenna arrays and diversity branches). $K = 2$, $L = 5$, $N = 511$ and $m = 1$

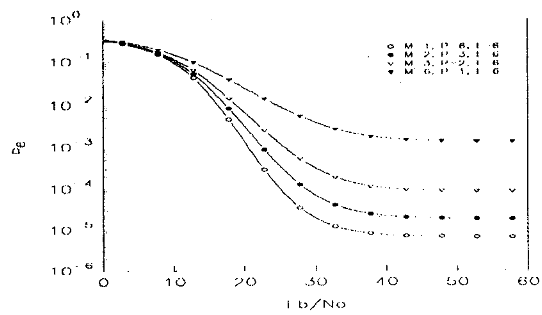

Fig. 7. Comparison on a basis of an equal number of antennas. $B = 6$, $K = 10$, $L = 6$, $N = 511$ and $m = 1$

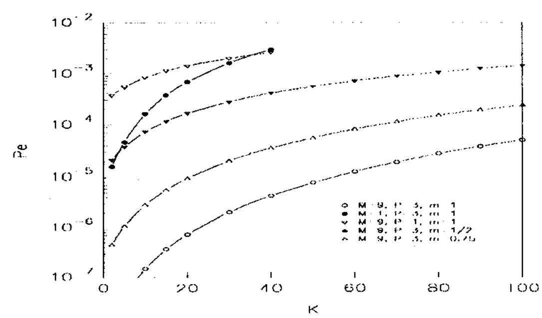

Fig. 8. Capacity increase when DSTF receiver are implemented as a function of the Nakagami parameter m. $L = 5$ and $N = 511$

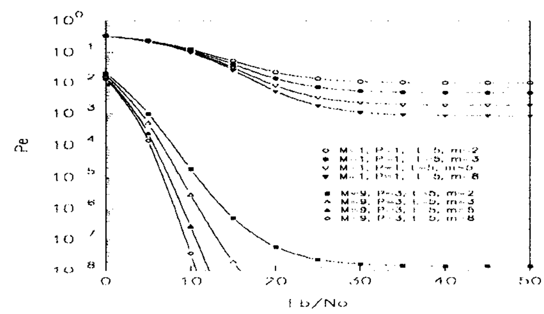

Fig. 9. Performance for high values (low fading) of the Nakagami fading parameter for $K = 50$, $L = 5$ and $B = \{1, 27\}$